

**History and Pedagogy of Mathematics
Americas Section, West Coast Meeting
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Tarski, Schools, and Geometry

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Joint work with Andrew and Joanna McFarland, of Płock, Poland.

Tarski, Schools, and Geometry

James T. Smith

◆ Who was Alfred Tarski?

- 1901–1939, Warsaw; 1942–1983, Berkeley.
- 1923–1953: perfected our framework for research in mathematical logic.
- 1953–1983: its preeminent figure.
- My teacher's teacher, my external PhD examiner.

◆ Biography

- Feferman & Feferman 2004

◆ Background

- Sznajder 2010 (presented at the March 2010 HPM meeting)

Tarski, Schools, and Geometry

- ◆ **Goals of this presentation**
 - **Sketch Tarski's upbringing and schooling in Poland, emphasizing the social context.**
 - **Depict Tarski's dual roles in geometry as mathematical researcher and schoolteacher.**
 - **Note where needed historical work is underway.**

Family & School

- ◆ In 1901 in Warsaw, Poland, then in Russia,
 - —about 685K, 30% Jewish—
- ◆ he was born *Alfred Teitelbaum*, a Jew.
- ◆ Stemming from prosperous families, in business,
 - his family lived near the city center.
- ◆ Secular and assimilated,
 - they spoke Polish at home.
- ◆ In maturity he complained bitterly about childhood antisemitic hazing.



Nation & High School (1)

- ◆ **August 1914: World War I began.**
- ◆ **January 1915:**
 - **Alfred → small, elite Szkoła Mazowiecka.**
 - **Faculty were highly educated.**
 - **Two became profs. at Warsaw.**
- ◆ **August 1915: Germans took Warsaw.**
- ◆ **February 1917—March 1918:**
 - **civil war, collapse in Russia.**



Nation & High School (2)

◆ **Summer 1918:**

- **Alfred & 6 others were graduated.**

◆ **Feferman & Feferman:**

His social identity had been transformed from that of a moderately assimilated bourgeois Jewish boy to that of a Polish patriot. He was neither religious nor a Zionist; politically his leanings were socialist. (2004, 16)



Nation & High School (3)

- ◆ **Fall 1915: Germans permitted U. of Warsaw to reopen as a new Polish university.**
 - **Topologist Kazimierz Kuratowski, then a student:**

...the [Warsaw] atmosphere...released a great creative potential...which produced a surprising development in many branches of science....
(1980, 28)

- ◆ **Fall 1918: Alfred entered the Univ. to study biology.**
 - **But it closed due to war's-end chaos.**
 - **We don't know what he did during 1918–1919.**

First University Year (1)

- ◆ **Kuratowski:**

One of the principal means...for [realizing the University's potential] was the concentration...in a relatively narrow field of mathematics...in which Polish mathematicians had common interests and...achievements which counted on a world scale. This field comprised set theory together with topology, and the foundations of mathematics together with mathematical logic. (1980, 31)

- ◆ **The University atmosphere was *electric*. Historian Andrzej Garlicki:**

...students could be met more often at political rallies and meetings than in university lecture rooms or laboratories. (1982, 341)

First University Year (2)

- ◆ Alfred reenrolled for 1919–1920 to study math, logic:
 - physics, sociology
 - Tadeusz Kotarbiński, logic & philosophy
 - Stefan Mazurkiewicz, calculus
 - Waław Sierpiński, set theory & measure theory
 - Stanisław Leńniewski, set theory & foundations seminar

- ◆ Strife, chaos: Poland fought 6 wars 1919–1922.
 - Math. profs. worked on decoding Russian messages.



Wacław Sierpiński
1882–1969



Stanisław Leśniewski
1886–1939

First University Year (4)

- ◆ **Summer 1920**
 - **Russians → Warsaw outskirts.**
 - **Not clear how Alfred was involved.**
 - **Decoding → total defeat of Russians.**

- ◆ **Fall 1920, Alfred returned to classes:**
 - **Mazurkiewicz, analysis**
 - **Sierpiński, algebra, set theory**
 - **Jan Łukasiewicz, logic**

- ◆ **Changed signature to *Tajtelbaum*.**

Taking Off (1)

◆ 1920–1922

- Alfred taught geometry at a girls' high school.
- Fired for being Jewish.

◆ 1922–1925

- Gave logic courses at National Pedagogical Institute.
- Taught math. at a *Jewish* girls' high school.

◆ Probabilist Mark Kac on Jews' academic employment:

...all doubts vanished if the candidate were to convert to Catholicism.

Polish anti-Semitism had always been largely religious. (1985, 28)

I doubt that, but that's what he thought.

◆ Spring 1922: Alfred converted to Catholicism.

◆ Fall 1923: Changed name to *Tarski*.

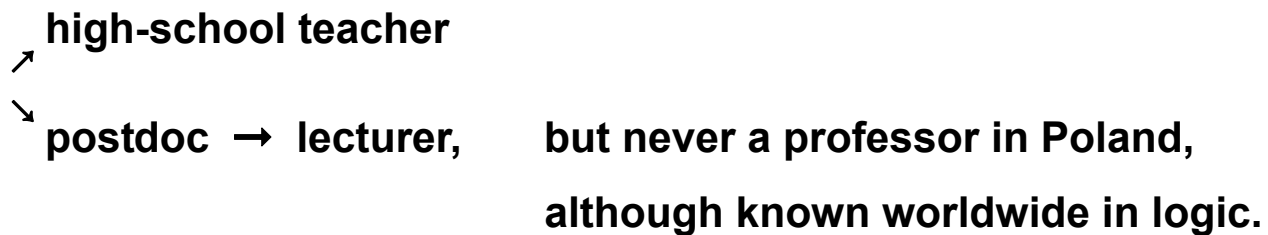
Taking Off (2)

◆ Spring 1924

- Tarski's PhD in logic under Leśniewski

◆ 1925–1939

- Taught geometry at the Stefan Żeromski school.

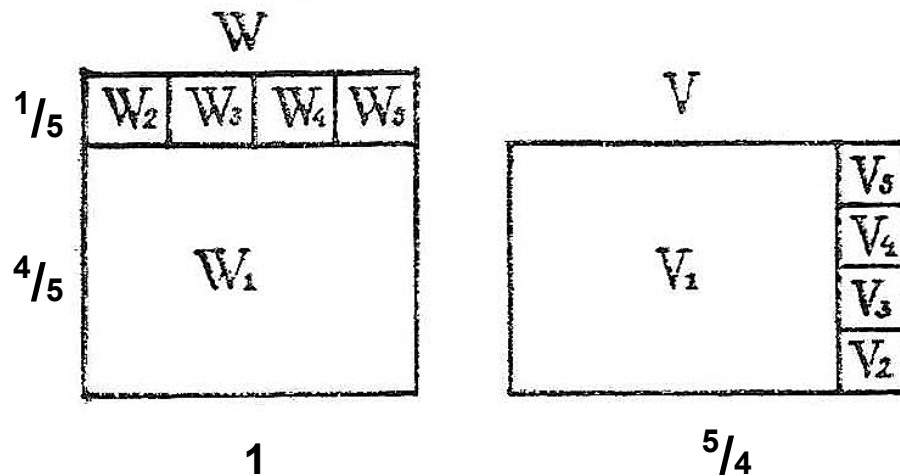
- Tarski  high-school teacher
postdoc → lecturer, but never a professor in Poland,
although known worldwide in logic.

◆ 5 long career threads emerged early:

- set theory, measure theory, teacher prep., geometry, logic.
- *Concentrate on some facets of first 4.*

Geometry (1)

- ◆ Preparing teachers, Tarski must have been thinking about this:
 - Polygons W, V have $=$ area \leftrightarrow *dissectible* into
= finite numbers of \cong polygons with disjoint interiors.



(Tarski 1931, fig 1)

- For polyhedra and volume, the same is not true.

Geometry (2)

- ◆ Powerful tools: Sierpiński's courses, Hausdorff 1914.
- ◆ Spring 1923:
 - Alfred met Stefan Banach, soon famous in analysis.
- ◆ They collaborated:
 - Banach 1923, *On the problem of measure*:
 - ∃ additive \cong -invariant extension of area to all bounded subsets of \mathbb{R}^2 (but not of volume in \mathbb{R}^3).
 - Tarski 1924, *On the equivalence of polygons*:
 - Polygons have = area \leftrightarrow *decomposable* into = finite numbers of \cong *disjoint* sets.
 - (Uses Banach 1923.)



Stefan Banach
1892–1945

- ◆ **Banach–Tarski 1924, *On decomposition of point sets into respectively congruent parts*:**
 - In \mathbb{R}^3 any 2 bounded sets with interior are decomposable into = finite numbers of disjoint \cong sets.



Banach–Tarski Paradox

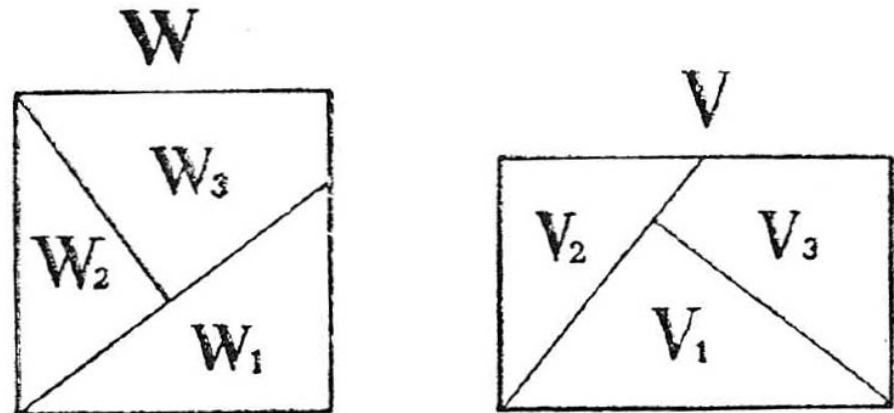
Set-theoretic decomposition doesn't correspond with physical or intuitive decomposition.

Teaching Geometry (1)

- ◆ 1931–1932 Tarski papers & one by Henryk Moese
 - in *Young Mathematician*, a teacher's journal,
 - on $\sigma(W,V)$, the degree of equivalence of 2 polygons with = area:
smallest number of \cong pieces required.

- ◆ Previous example: W, V with 5 parts

- Look: $\sigma(W,V) \leq 3$.
- Can you cut them to prove $\sigma(W,V) = 2$?



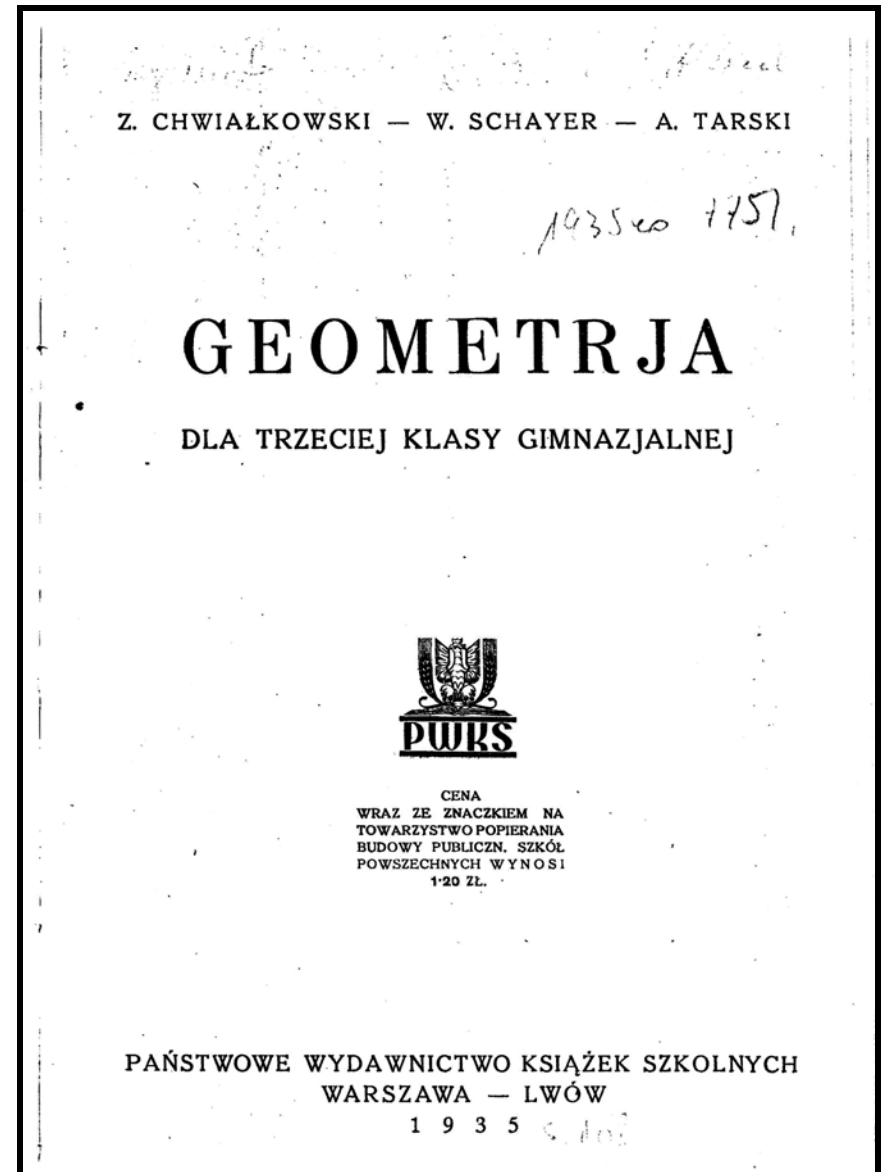
(Tarski 1931, fig 2)

- ◆ Tarski invited others to pursue this subject,
 - but that never happened.

Teaching Geometry (2)

◆ 1935

- Coauthored high-school text
- We know little about the others.
- Nor how it was used.
- Last chapters, on area
- *We're working on it!*



Teaching Geometry (3)

◆ Tarski's 1934-1938 high-school student

Witold Kozlowski:

- gifted students would visit Tarski's home.
- Tarski's Univ. students
→ his school lectures.
- Tarski's favorite subject was
area measure.

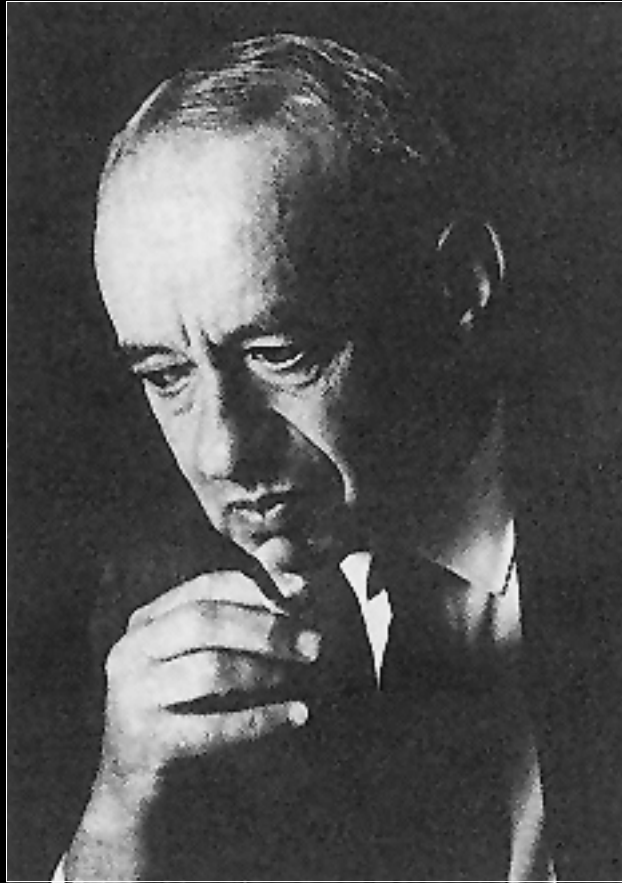


Joanna & Andrew McFarland

Witold Kozlowski (1919–)

Conclusion

- ◆ Tarski, on a lecture trip, was stranded in the U.S. when the Nazis invaded Poland.
- ◆ Nazis murdered his entire extended family,
 - except his wife (not a Jew) and their son and daughter.
- ◆ He presented his favorite subject—degree of equivalence—to Berkeley schoolkids,
 - and after my thesis defense in Regina in 1970,
 - and connected it to results in undecidability!
- ◆ Tarski's pubs. mentioned here will be translated in an edition by the McFarlands and me.



Alfred Tarski
1901–1983

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Thank you for your interest!

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Tarski, Schools, and Geometry: References

James T. Smith

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Geometry Sally & Sally 2007, chapter 5.
History Davies 1982, Sznajder 2010

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Illustrations

1. Alfred in 1912 from Feferman & Feferman 2004.
2. Szkoła Mazowiecka in 1915 from Jadacki 2003b.
3. Alfred in 1918 from Givant 1991.
4. Sierpiński from Garlicki 1982.
5. Leśniewski from Givant 1999.
6. Dissection from Tarski 1931.
7. Banach from Jakimowicz & Miranowicz 2007.
8. Another dissection from Tarski 1931.
9. *Geometria* scanned by McFarland.
10. McFarlands, Kozłowski in 2010 by Anna Kozłowska.
11. Tarski in 1966 from Givant 1991.