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Point-hyperplane axioms for orthogonal geometries.
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The author presented in 1973 a theory of orthogonal geometries—incidence geometries with an orthogonality relation—that admits Euclidean, hyperbolic and elliptic models of all dimensions, even infinite. The purpose of that work was to provide a framework for formulating a theory of reflections that would extend to arbitrary, even infinite, dimensionality the notion of a metric geometry, studied by F. Bachmannand his colleagues. However, only one aspect of Bachmann's work was fully extended within the orthogonal geometry framework; axioms were formulated by the author for metric geometries of arbitrary dimension. Their groups of motions were then constructed and related to the orthogonal groups.

The second aspect of Bachmann's program, constructing a geometry from the generators of a special kind of group that will serve as its group of motions, is generated by the reflections in points and in hyperplanes, termed orthocomplemented, that have perpendiculars. Commutativity of these reflections corresponds to incidence and orthogonality relations.

The goal of this paper is to present a set of axioms with point, orthocomplemented hyperplane and orthogonality as primitive notions, in such a way that the original axioms for orthogonal geometries are satisfied. The new set of axioms also applies to the elliptic case.

The author lists four axioms, H1–H4, which are satisfied by all orthocomplemented hyperplanes of an orthogonal geometry $(r, G, E, \perp)$. From points and hyperplanes satisfying axioms H1–H3 are constructed dual elliptic geometries that behave like bundles of orthocomplemented hyperplanes. These geometries are "inverted", obtaining the local elliptic geometries consisting of bundles of lines. These then used to connect points and define planes. The author demonstrates the main theorem which states that the above process meets the goal.

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