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Metric geometries of arbitrary dimension.
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From the author's introduction: "Metric geometry is the study of structures that satisfy Hilbert's incidence axioms and some simple orthogonality conditions, admit reflections in all points and lines, and satisfy the three-reflections principles. Axioms for metric geometry can be phrased in terms of "geometric" objects and relations—for example, points, lines, planes, and orthogonality—or in terms of the group of motions. The first approach is called synthetic; the second, group-theoretic."

Group-theoretic foundations for metric geometry have been given for the plane case by F. Bachmann [*Aufbau der Geometrie aus dem Spiegelungsbegriff*, Kapitel II, Springer, Berlin, 1959; MR0107835; Russian translation, Izdat. "Nauka", Moscow, 1969; MR0248597], for the three-dimensional case by J. Ahrens [Math. Z. **71** (1959), 154–185; MR0108752], for the arbitrary finite-dimensional case by H. Kinder ["Begründung der n-dimensionalen absoluten Geometrie aus dem Spielungsbegriff", Dissertation, Christian-Albrechts-Univ., Kiel, 1966], and for arbitrary dimension by G. Ewald [Abh. Math. Sem. Univ. Hamburg **41** (1974), 224–251; MR0348605]. As regards the synthetic approach, foundations for metric geometry have been given for the plane case by Bachmann [op. cit., § 2], for the three-dimensional case by H. Scherf ["Begründung der hyper-bolischen Geometrie des Raumes", Dissertation, Kapitel I, Christian-Albrechts-Univ., Kiel, 1961], for arbitrary finite dimension but only in the elliptic case by Kinder [Arch. Math. (Basel) **21** (1970), 515–527; MR0281091] and by Kinder and H. Wolff [Abh. Math. Sem. Univ. Hamburg **34** (1969/70), 252–265; MR0268763]. In this paper the author gives the synthetic foundation for metric geometry of arbitrary dimension. The undefined geometric notions are just those mentioned in the introduction. The axioms are divided into four groups. The first group consists of Hilbert's incidence axioms, modified to admit arbitrary dimensionality. The second group consists of H. Lenz's orthogonality axioms [Math. Ann. **146** (1962), 369–374, especially § 1; MR0139033], modified to admit elliptic models. The third group postulates the existence of reflections in all points and lines. The fourth group consists of the three-reflections principles. The author's main result is a representation theorem, formulated separately for the elliptic, hyperbolic, and parabolic cases. Each model that occurs is determined by a metric sub-domain of the analytic projective or affine metric space over a commutative field of characteristic different from 2. In the appendix the author claims that, by comparing the models, in the finite dimensional case his theory is equivalent to that of Kinder (see the third reference cited above). But the notion of "equivalence" implied in this claim is not determined (this is not an easy task because the two theories under discussion differ so much in their character) and so this statement should be understood in a somewhat intuitive way. At the end the author verifies all of Kinder's axioms in his own theory, but in this way he gets only the interpretability of the first theory in the other. No comparison is made with Ewald's theory, which, however, has not yet appeared.