AMERICAN MATHEMATICAL SOCIETY
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Citations From References: 0 From Reviews: 0

MR779181 (86g:51035) 51F20 51D99 51F10
Smith, James T. (1-SFSU)
Group theoretic characterization of metric geometries of arbitrary dimension.
Geom. Dedicata **17** (1985), *no. 3,* 287–296.

The earlier work of the author extended to arbitrary, even infinite, dimensions, the work of F. Bachmannand his colleagues on the foundations of metric (absolute) geometry. However, only one aspect was fully extended. Bachmann's program has two phases. In the first—the synthetic phase—an axiom system is developed which involves the notions of incidence and orthogonality and the existence and properties of reflections in various subspaces. These reflections generate a group of motions, which is then related, via projective geometry, to the orthogonal group of linear algebra. In the second phase, axioms are developed to characterize these groups of motions: from a group containing generators satisfying certain axioms a geometry is constructed whose group of motions is isomorphic with the given group. In the present paper a group-theoretic foundation is given for metric geometries in general, of arbitrary dimension.

An orthogonal geometry is a geometry of points, lines and planes with an orthogonality relation on its lines, that satisfies various incidence and orthogonality axioms. Since the axiom system of the present paper involves groups generated by elements that behave like the reflections in the points and orthocomplemented hyperplanes of a metric geometry, the construction of a geometry from the group is facilitated if we can construct an orthogonal geometry from its points and orthocomplemented hyperplanes. This construction was given by the author in a previous paper [see the preceding review] with the warning that in an infinite-dimensional geometry not all hyperplanes have reflection—just the orthocomplemented ones, i.e., those that have orthogonal lines. The present paper brings to light the lattice-theoretic basis of higher-dimensional metric geometry.

The author gives seven axioms and axioms $8_1, 8_2, 8_3, \cdots$, which state necessary and sufficient conditions for there to exist an isomorphism from a group $\mathcal{B}$ to the group of motions of some metric geometry, and under this isomorphism the members of some subsets of $\mathcal{B}$ correspond to the reflections in the points and orthocomplemented hyperplanes.

From a given model of axioms 1–5 is constructed an orthogonal geometry, satisfying the point-hyperplane axioms of the previous paper. This geometry is called the group geometry, which is a metric geometry whose group of motions is isomorphic to the inner automorphisms of $\mathcal{B}$.

The main task of this paper is to show that any group satisfying axioms $1–7, 8_1, 8_2, \cdots$ is isomorphic to the group of motions of its group geometry. The following main result is obtained: A group $\mathcal{B}$ generated by the union of sets $\mathcal{R}$ and $\mathcal{S}$, each consisting of involutions and invariant under all automorphisms of $\mathcal{B}$, satisfies axioms $1–7, 8_1, 8_2, \cdots$ if and only if there is an isomorphism from $\mathcal{B}$ to the group of motions of a metric geometry, and under this isomorphism the members of $\mathcal{R}$ and $\mathcal{S}$ correspond to the reflections in the points and orthocomplemented hyperplanes. *Dov Avishalom*