

## Thesis Outline

### I. Introduction

#### A. General Linear Group

1. Discuss  $GL_n$  and expand notion to the limit group

#### B. Automorphisms of the Formal Power Series Ring

1. Define a formal power series ring
2. Discuss what automorphisms of a formal power series ring look like

#### C. Discuss mappings from General Linear Group to the Group of Automorphisms of the Formal Power Series Ring and Vice Versa

### II. Motivation

#### A. Introduce the abelianization of a group

#### B. Discuss the role of the commutator subgroup in the abelianization of groups

#### C. Briefly introduce Whitehead's Lemma, by which we know what the commutator subgroup of the general linear group is

#### D. Introduce motivating question

1. Since there are mappings from one group to another, is the abelianization of the General Linear Group the same as that of the Group of Automorphisms? (i.e. Are they isomorphic?)
2. Discuss the characterizations of the commutator subgroup already known due to Dr. Gubeladze's article
  - a. Power series rings with coefficients from ring of prime characteristic  $p \geq 5$
3. Introduce our goal: Characterize commutator subgroups for rings of characteristic 2 and 3

### III. ?? Body

#### A. Introduce the notion of the truncated ring and its automorphisms

#### B. Introduce the notion of the truncated ring of unmixed automorphisms (very specialized subgroup of larger group)

1. Introduce "nice" matrix theoretical formulas for compositions and commutators that we get when looking at truncated ring

#### C. Show how "nice" matrix theoretical formulas give us a non-computational way of looking at abelianization

1. Homomorphism from one matrix to a "rearranged" version of the matrix
2. ?? K-Theoretical point of view ?? (More research necessary on my part to understand this part of project)
  - a. ?? A few short words  $K_1$  and how we will basically view it as the abelianization of our group??
  - b. Define new group homomorphism from  $K_1(R)$  to the group of units
  - c. Show that the above homomorphism splits
  - d. ?? Introduce  $SK_1(R)$  and introduce (prove???) facts?? (Need to understand more)
    - i. If  $R$  is a field, PID, local ring, then  $SK_1(R) = 0$
    - ii. If we have a nilpotent ideal, the  $SK_1(R) = SK_1(R/I)$
    - iii.

#### D. Setbacks

1. Cannot be applied to rings containing a ring with prime characteristic 2  
(because we used fact that  $a^2 = a$  in our argument)

#### IV. Further applications

##### A. ??Similar argument for coefficients from $Z_3$ ??

1. More research needs to be done, but likely to hold true
2. ?? Do computational part for  $Z_3$  if time allows

##### B. Different subgroups

1. automorphisms with only linear part and a fixed “prime” power
2. ?? What would studying this subgroup do for us??

#### V. Conclusion

#### VI. References