1. INTRODUCTION

This booklet offers suggestions on preparing manuscripts for the journals of The Mathematical Association of America. Most of the advice applies to books as well, and one or two remarks refer specifically to books; but if you are planning a book, be sure to consult Halmos [2], particularly Sections 4, 6, and 7.

The publications of the Mathematical Association are **expository** publications, dedicated to the advancement of college mathematics. Unlike some research publications, ours are responsible first to our readers, then to the authors; but much of the advice is appropriate to research articles as well.

Most of the suggestions are simple matters of common sense; some involve judgments on which opinion is divided; and others are just plain my own prejudices. The style is largely *Do this* and *Don't do that*, so the suggestions come across rather as rules. Each one has its counterexamples. But if you do break a rule, do it deliberately, not from carelessness.

Articles for MAA journals are expected to be well motivated and of wide interest. The exposition should be clear and lively, and in fact the quality of exposition counts as much as mathematical content in determining whether a paper will be accepted for publication.

Articles showing a new slant on a familiar result are welcome; but don't merely reinvent the wheel in a well-known mode. Before you write up your results, be sure to search the literature and check with your colleagues. Many manuscripts are rejected out of hand because the author had not taken the trouble to consult even standard texts to see what was already available in print.

THE AMERICAN MATHEMATICAL MONTHLY is directed to readers in the range of levels from advanced undergraduate to professorial; think of the "average" reader as having had a year or more of graduate work. MATHEMATICS MAGAZINE and THE COLLEGE MATHEMATICS JOURNAL are devoted to undergraduate mathematics—with the JOURNAL concentrating on the first two years—and many of the readers are students; think of an article as supplementary material for a standard course.

To whom, then, is your article addressed? Keep a specific group in mind—or, better, a specific person: how would you explain your results to your colleague down the hall, or to the student you were chatting with just now?

Books. The Association publishes books at several levels. The MAA STUDIES IN MATHEMATICS series describes recent research. Each book is devoted to a single subject and contains expository articles from several contributors, written at the collegiate or graduate level. The CARUS MATHEMATICAL MONOGRAPHS are intended to make topics in pure and applied mathematics accessible to teachers and students of mathematics, and also to nonspecialists and scientific workers in other fields. The DOLCIANI MATHEMATICAL EXPOSITIONS are designed to appeal to a broad audience, challenging the talented high school student and intriguing the more advanced mathematician; topics thus far have emphasized elementary combinatorics, number theory, and geometry. The NEW MATHEMATICAL LIBRARY is a series of paperbacks addressed to college and high school students who are interested in understanding and appreciating important mathematical concepts beyond those featured in traditional mathematics courses. The Association is always looking for good books, whether they fit into one of these series or not.

2. GENERAL ORGANIZATION

2.1. The title

Keep your title short and include key words to make it informative; a long title sounds pompous and is a nuisance to refer to.

On a theorem of David Copperfield

is short but uninformative.

Algebraic solutions of linear partial differential equations

encroaches on the limit for length but includes key words and is good.

Steer clear of symbols: they will cause typesetting problems in the original article and thereafter in bibliographies and wherever else the title is quoted.

2.2. The introduction

The first paragraph of the introduction should be comprehensible to any mathematician. Describe in general terms what the paper is about; and do it in a way that entices the reader to continue reading. Settle for a rough statement in words; eschew a precise statement loaded with symbols and technical terms.

Your very first sentence, in particular, must command the reader's immediate interest.

Would you guess that most continuous functions are nowhere differentiable?

is excellent.

This paper describes an unusual application of the Mean-Value Theorem

is also good; it could say more about the application, but can stand as is because the Mean-Value Theorem is so familiar.

Consider a sheaf of germs of holomorphic functions

is too technical (even in the late 1980s).

$$\textit{Let } f(x) = \int_0^x \arccos t \ \arcsin 2t \, dt$$

is totally unmotivated and is atrocious.

2.3. Prerequisites

It helps to mention any unusual prerequisites and to review a few key definitions that might not be well known—with references. But don't do a snow job. If you feel the need for an entire page, you should stop and rethink the whole article; remember that the reader does not have to be familiar with each theorem you are going to invoke but need only be able to understand what it means when you quote it. In any case, you don't have to list everything all at once: your paper will be friendlier if you weave the information unobtrusively into the text as you go along.

2.4. Notation and terminology

You will certainly require some terms and symbols. Decide which ones you really need. Most mathematical articles (and lectures) use too many. (*Proof:* Have you ever encountered one with too few?) I once heard a very famous mathematician begin a talk by introducing two dozen symbols, one of which was so obviously unnecessary that I kept watch to see whether he would ever use it. Sure enough, 47 minutes later he wrote the symbol for the first and only time. He did have the good sense to recall its definition—by the same stroke confirming that the symbol should never have been introduced at all.

An introductory section listing every term and every symbol that are going to come up may serve as a handy reference, but will overwhelm the reader. Again, it is better to introduce such things in small doses. If you still feel a strong need to include a glossary, put it at the end.

Chapter 5 and the Appendix offer further suggestions about symbols.

2.5. Organization and pace

If your paper runs to several pages, divide it into sections with informative titles. This will help clarify the logical structure of the paper and make it easier to follow.

Introduce one idea at a time. Make liberal use of examples, perhaps suppressing the most abstract formulation of the idea. Link ideas to what you assume is familiar to the reader.

2.6. The bibliography

The purpose of the bibliography is to alert readers to the existence of books and articles that give history, priority, and attribution. List only published works, therefore, and make the list complete—don't just jot down a few references as an afterthought. Never mind whether a work is unavailable at most college libraries: that's irrelevant.

Private communications should be woven into the text, not dignified with a number leading to useless flipping of pages. Likewise, "to appear" should not appear, unless the paper is already accepted by a journal that can be named.

2.7. The index (to a book)

The index is supposed to help the reader. It should therefore be something more than a permutation of the book into alphabetical order.

The big job is selecting and classifying. This requires a feeling for the relative importance of the topics and hence can be done properly only by you. For example, in a book that quotes the axiom of choice, the index typically refers to the one place it is quoted. But if a central purpose of your work is to study the role of the axiom, then you might well decide to refer to every proof that invokes it. You can start this organizational work as soon as the manuscript is finished: it does not depend on knowing the page numbers.

Don't ask the reader to guess what name you have listed a topic under; instead, *you* try to guess which name the typical reader will look for. If there are several, list them all. I have before me an index that references the Cauchy-Schwarz-Buniakowsky inequality six times: each of the names appears twice, once in the main list and once under "inequalities" [20]. Excellent! Always err in the direction of helpfulness.

3. PRESENTING YOUR RESULTS

3.1. State first, prove second

Keep in mind that you have to maintain the reader's interest at all times. An easy way to lose it is by stringing along a succession of statements with no discernible goal—the classic example being a sequence of arguments culminating in the triumphant cry: "We have proved the following theorem."

RULE 42 (the oldest rule in the book) [**26**]:

Always state the theorem before proving it.

(This has some pleasant exceptions.)

3.2. Generality

It is good research practice to analyze an argument by breaking it into a succession of lemmas, each stated in maximum generality; it is poor expository practice to publish the results that way. Develop the argument in the special case that applies to your paper. If you want to mention the generalization for the record, don't state it as a formal theorem but as a passing remark; and postpone it until after you have stated and proved the special case. Do tell the reader, though, whether the general proof is a straightforward extension of the one you gave or requires new ideas; and mention where to find it.

3.3. Style

State theorems concisely. Try for one short sentence—or at most two, one for the hypothesis and one for the conclusion.

EXCEPTION. It is occasionally convenient to build modestly on related ideas, as in the form:

Assume P. Then Q. If also R, then S.

The statement of a theorem is not the place for definitions, proofs, or other discussion.

EXCEPTION: If R is an immediate consequence of Q, then "discussion" at the level of

If P, then Q; hence R

is acceptable, and in fact helps keep the organization crisp.

"Concise" does not mean short, but only that you not waste words. The common style that starts with "The following four conditions are equivalent", and follows with a list, is undeniably concise; it is acceptable no matter how the sentence count comes out. But I would balk at eleven equivalent conditions, because at least six of them will surely be merely minor variants of the others. Confine the theorem to the basic list and relegate the others to an accompanying remark.

The statement of a theorem is generally the wrong place to introduce notation for the proof:

A differentiable function f is continuous. **X**
A differentiable function is continuous. √

The first way is underhanded. (If not followed by a proof, it is gross.)

Both versions illustrate a convention about quantifiers that we tend to withhold from our beginning students, whom we then scold for not getting it right: saying "a" (or "any") differentiable function to mean "every" differentiable function. Why not be safe and say "every"? Another common and acceptable form is, of course:

Differentiable functions are continuous √

—though I suppose an expert nitpicker could ask whether that means some or all. This way is at least safe from symbolitis: no one (I hope) would write:

Differentiable functions f are continuous. **X**

3.4. The character of a proof

It is often good to illustrate the theorem before proving it—or even instead of proving it, in case the illustration contains the essence of the proof, or the proof is too hard.

Strive for proofs that are conceptual rather than computational—the way you would describe the result to a colleague (or student) during a walk across the campus. If a proof is at all involved, begin by explaining the underlying idea. Leave all purely routine computations (i.e., not involving an unexpected trick) to the reader.

Use a picture, perhaps as an important step *in the proof itself*. MATHEMATICS MAGAZINE has for several years been encouraging this spirit with its delightful series entitled *Proof without words.*

J. E. Littlewood, in his classic treatise [3], presents these ideas in telling style, first concocting a barbaric proof (of the Weierstrass approximation theorem), then presenting a civilized one *with the help of a picture* [pp. 30-36].

3.5. Keeping the reader informed

Keep the reader informed of what you are doing and of how things stand. I have always enjoyed reading Sierpinski: first he tells you what he is going to do, then he does it, then he tells you he did it.

If you use a key term or symbol after a long spell without it, recall its definition or refer to where it was introduced. Your readers will be grateful.

Make sure the reader knows the *status* of every assertion you make—whether it is a conjecture, the theorem just proved, something from the reader's background, a well-known result, a proposition you are about to establish. The last is the most important. Avoid "roadblocks". If you write:

But G is abelian. To see this, consider . . . ,

the reader is tempted to stop at the period and start thumbing back in search of the proof. It is better to say:

But G is abelian, as we now show . . .

—since the reader is likely to keep going past the comma. For a more substantial improvement, put the hint of the forthcoming proof at the beginning:

Next, G is abelian.

And why be parsimonious? Put in three additional words and say:

Next we show that G is abelian.

Now the flow is so smooth the reader won't even realize there was a problem.

Tell the reader when a proof has ended. The best way is to say so outright: "This completes the proof", or "□" or "♦". (CAUTION: Some editors may disallow a symbol as too formal-looking.) An alternative is to say so indirectly—e.g., start the next paragraph with "It follows from this theorem that . . ."

3.6. Citing reasons

Cite supporting reasons informatively.

When you invoke a key hypothesis, identify it as such and tell which one it

is. Suppose for example your hypotheses are:

$a = 2$, f is continuous, and G is abelian,

and at some step in the proof you need just the second of these. Don't just say:

By hypothesis, we have . . . ;

say:

By hypothesis, f is continuous; hence we have . . . .

This helps both you and the reader see what is making things tick.

Every development has mountains and valleys. Name the mountains and refer to the names. Say "By the Fundamental Theorem of Calculus", not "By Theorem 5.7". Say "By the triangle inequality", not "By Axiom 3".

3.7. Citing references

When you quote a proposition from another work, tell whether it is hard or easy; this helps readers decide whether to read on without wasting time or try working it out for themselves. If the result is relatively unfamiliar, include a citation:

According to a deep theorem of Gulliver [**31**, p. 56], *every such Lilliputian space is uncountable*

—and the reader can decide peacefully whether to continue reading or go look up Gulliver's proof.

In case Gulliver's phrasing is much more general than yours, insert a hint on how to perform the transition:

(His theorem is stated in terms of arbitrary lattices and semi-Lilliputian spaces; to apply it here, note that our family of open sets forms a lattice, and that Lilliputian spaces are semi--Lilliputian.)

3.8. Proofs by contradiction

I classify contradictions in proofs as *essential*, *questionable*, or *spurious*. In the familiar proof that $\sqrt{2}$ is irrational, the contradiction is essential: you assume the contrary, and you use that assumption in an essential way in the course of the proof.

As an example of a questionable contradiction, consider the following proof that if a^2 is even, then a is even.

> *Assume on the contrary that a is odd.*
>
> * *Then a is of the form $2b+1$, and a computation*
> * *shows that a^2 is of the same form and hence is also odd.*
>
> *This contradicts the hypothesis; therefore we must reject the assumption that a is odd. Hence a is even.*

My concern is that the equivalence of a proposition with its contrapositive is something we all take for granted—the two forms are often interchanged as a matter of course—and I see no reason to include yet another proof. Thus, I prefer to say that I will use the equivalent form—if a is odd then a^2 is odd—and then prove that assertion directly (as is done in the marked lines).

A proof by spurious contradiction sets up and knocks down a straw man. As an example, consider the following proof (which I have seen in print) that the functions e^x and e^{2x} are linearly independent over the reals.

> *Assume on the contrary that e^x and e^{2x} are linearly dependent. Then there exist constants c_1 and c_2, not both zero, such that*
>
> * $$c_1e^x + c_2e^{2x} = 0 \quad \textit{(for all } x\textit{)}.$$
>
> * *Then $c_1 + c_2e^x = 0$. Differentiating, we get $c_2e^x = 0$,*
> * whence $c_2 = 0$. *Then* $c_1 = 0$. *Thus,* $c_1 = c_2 = 0$.
>
> *This contradicts the fact that c_1 and c_2 are not both zero; therefore we must reject the assumption that the functions are linearly dependent. Consequently, they are linearly independent.*

The point is that while the proof assumes that c_1 and c_2 are not both zero, no use is ever made of that assumption (except to knock down the straw man). To see this, delete everything but the marked lines (and begin the first of

them with "Assume that"). You now have a direct proof of linear independence.

No one would start a proof of the formula $x^2 - 1 = (x-1)(x+1)$ by writing:

> *Assume the contrary. Then there is a number x_0 such that*
>
> $$x_0{}^2 - 1 \neq (x_0 - 1)(x_0 + 1).$$

But when the setting is slightly more involved, that construction is harder to notice. When you first search for a proof, by all means begin if you wish by assuming the conclusion false. But at some stage before sending in your article, check your proofs for spurious contradictions.

4. MATHEMATICAL ENGLISH

This chapter contains suggestions about the use of the English language in mathematical writing. Some discussion of English usage in general is given in Chapter 6.

4.1. We and I

The use of "we" to mean you and the reader, or you with the reader looking on, is universal; to avoid gruffness, it is virtually indispensable:

> *We see from* (1) *that . . . .   Let us now prove that . . . .*

Joint authors have to think carefully, then, about how to refer to themselves:

> *We proved in* [31] *that . . . . (?)*

Safer:

> *The authors proved in* [31] *that . . . .*

People disagree about whether an individual author may use "I" without sounding egotistical, or "the author" without sounding pompous. What counts is the spirit in which you write. If you need four references to yourself in the same paragraph, combine them into one:

"These results are based on my papers [**31,32,33,34**]";

then proceed. You are still permitted crucial references as they come up, such as "[32, Theorem 6]".

In the hands of the right person, "I" can be downright refreshing. My all-time favorite is a footnote in a 1950 paper by Paul Erdös [**21**, p. 137]:

This remark is probably due to Sierpinski, but I do not remember for sure.

What could be more warm and human? Try substituting "we" or "the author", or some circumlocution.

Don't use "myself" to mean simply *me*; reserve it for emphasis:

These results were obtained jointly by Hilbert and myself. **X**
These results were obtained jointly by Hilbert and me. √
I made the same error myself. √

4.2. The grammar of symbols

A symbol represents a word or phrase. Writers do not always agree about whether it can represent more than one. My attitude here is permissive. I think it is all right to use "=" to mean either *is equal to*, just plain *equal to* (with no verb), *be equal to*, or *which is equal to*:

We see that $x = 0$.
Take $x = 0$.
Let $x = 0$.
Then $x^2 + y^2 = r^2 = 1$.

These expressions read smoothly, because the introductory words, "We see that", "Take", and "Let" in the first three, and the sequence in the fourth, prepare you for what is coming. I use "∈", "<", and other symbols in the same free way.

On the other hand, I would balk at *were equal to* as in:

If $x = 0$, *then we would have* . . . , *(?)*

because (in the absence of a prefatory hint) the cue for how to read the equals sign to fit the grammar does not appear until after you have read it wrong. Solution: say it in words; or stick to simple tenses.

Finally, *never* suppress *which* after a conditional—e.g., don't use

If $a = b = c$ . . .

to mean

If $a = b$, *which is equal to* c, ...

Thus:

Let $\delta = \frac{3}{4}\varepsilon > 0$. *Then* . . . . **X**

Let $\delta = \frac{3}{4}\varepsilon$. *Then* $\delta > 0$, *and* . . . . √

Punctuate displays. A sentence that ends with a displayed formula still requires a period. (Some people, notably commercial book publishers, disagree.)

4.3. *If* . . . , *then* . . .

I once got a paper to referee that stated a theorem in the following form:

If $a = b$, $c = d$, $e = f$, $g = h$. *(?)*

Include the "then"—and the comma preceding it—as a matter of habit. Do this even in the simplest case:

If $x > 0$, *then* $\log x$ *is defined*; √

your reader is seeking information, not mental calisthenics. (For an exception to the rule about the comma, see the last two lines of the proof in the Appendix.)

Don't ask a historical fact to depend on a mathematical hypothesis:

If $x > 0$, *then Euler proved in* 1756 *that* . . . . **X**
Euler proved in 1756 *that if* $x > 0$, *then* . . . . √

Use "then", not "therefore", after an assumption:

Suppose I lend you $10. Therefore you owe me $10. **X**
Assume $x = 3$. *Therefore* $2x = 6$. **X**
Assume $x = 3$. *Then* $2x = 6$. √

The construction

Since . . ., then . . . **X**

is discordant. Instead of "then", say "it follows that", or "we have", or nothing:

Since this limit exists, then the series converges. **X**
Since this limit exists, the series converges. √

4.4. Definitions

Don't use "if and only if" in a definition (except in formal logic)—it's too pompous. By tradition, "if" is sufficient:

An integer > 1 *is said to be prime if its only positive divisors are itself and* 1. √

"Let" after the title DEFINITION is redundant and sounds gauche:

Let $|S|$ *denote the cardinal number of* S. √
DEFINITION. *Let* $|S|$ *denote the cardinal number of* S. **X**
DEFINITION. $|S|$ *denotes the cardinal number of* S. √

5. SYMBOLS

5.1. You can do with fewer than you think

We mathematicians are trained in the use of symbols and tend to put them in liberally. That's not necessarily bad in itself—there they are in case we need them. What we have to remember later on is to take out those we don't need—and simplify any involved ones that remain. The Appendix presents a case study on the use of symbols, with examples of both types of editing.

In an extreme case, a symbol can be removed the instant it is put in:

Obviously, every group G *of prime order is simple.* **X**
Obviously, every group of prime order is simple. √

Suppose you really "need" a whole menagerie of symbols—what then? Answer: your masterpiece does not qualify as an expository article. Rethink it and rewrite it. You may well end up simplifying the mathematical argument as well. The Appendix demonstrates a good example of that too.

Sometimes one introduces a symbol for the sole purpose of reducing clutter; the question then is whether the exchange was advantageous. If $x^2 + x + 1$ shows up casually in the course of a computation, you may be tempted to call it u, or $p(x)$, in order to cut down on clutter. I would say don't, even if it appears eight times: you would be asking the reader to keep track of something ephemeral in exchange for a saving of little consequence. On the other hand, if you are going to refer three or more times to a particular 4×4 matrix, or to an expression like

$$A_1^{-1} A_2^{-1} \cdots A_n^{-1},$$

then I would say yes, display it and assign it a letter, or number it in the margin.

Extra generality often requires extra symbols. Consider replacing a general formulation by an illustrative example.

Extra generality often requires fewer symbols. Consider replacing a specialized formulation by a simpler, more general one—with a probable gain in insight. For example, if you believe that two points determine a line even when the equation of the line is not written down, then you can derive the Mean-Valıe Theorem (from Rolle's Theorem) without bringing in the equation

$$y = f(a) + m(x - a),$$

ınd certainly without calling up the detailed formula

$$F(x) = f(x) - f(a) - \frac{f(b) - f(a)}{b - a}(x - a).$$

Iere f is continuous on $[a, b]$ and differentiable on its interior. Let g be any unction that has the same properties and agrees with f at a and at b. Then $- g$ satisfies the hypotheses of Rolle's Theorem, and there is a point z in $a, b)$ such that $f'(z) = g'(z)$. The Mean-Value Theorem is the particular case in vhich the graph of g is a straight-line [22].

5.2. Choice of notation

Use standard or familiar or suggestive notation that your readers can assimiate easily, allowing them to devote their energies to the mathematics.

Use *consistent* notation. This may take some planning. No one would call he angles of a triangle δ, L, and t_1. So don't get stuck with

$$a_1x + a_2y, \quad \text{or} \quad ax_1 + bx_2: \quad (?)$$

ırepare the notation so that you end up with

$$ax + by \quad \text{or} \quad a_1x_1 + a_2x_2. \quad \surd$$

Mathematicians use a variety of alphabets, and we have developed a number of helpful typographical conventions, such as the standard symbols e (italic), **R** (bold), π (Greek), Σ (stylized Greek), Ø (Danish), ℵ (Hebrew). All hese are at your disposal. But use them judiciously. Don't go out of your way o use lots of different fonts and sizes—that's an unnecessary invitation to typeetting errors.

Finally, don't forget to review the symbols you have assigned—it is not ıncommon to discover that the same one has been used in more than one way.

5.3. Look

Pay attention to the *visual* impact of the printed page. A long stretch of symbols is hard to read. Break it up with connectives. Phrases such as

It follows that
By hypothesis
On the other hand, from (41), *we know that*

are friendly and allow the reader to relax. If they seem too wordy, say "Hence", "Now", "But".

After a lengthy string of computations, start a new paragraph, with some introductory text.

Be sparing in the use of "∃" (except in formal logic), and don't overdo "iff": write out the words. (Save the abbreviations for the blackboard when you are talking faster than you—or your students taking notes—can write.)

5.4. Mixing symbols with text

Devote some thought to how you use symbols within a line of text.

Then every number on the left < *every one on the right* (?)

and

We conclude that the two expressions are = (?)

are grammatically impeccable, but most people would agree that they are inelegant at best. Editors will reject both.

Editors strongly discourage starting a sentence with a symbol, as it can be counted on to confuse the copy editor, the compositor, and the reader. I myself recognize exceptions—for example, in a sentence displayed by itself. My irrevocable rule is: *Never start a sentence with a symbol when the preceding sentence ended with a symbol:*

Then $a > 4$. $b > 4$ *also, since* . . . **X**

5.5. Your numbering system

If your manuscript is more than 8 typed pages (double spaced), divide it into numbered sections and number theorems serially within each section: 1.1, 1.2, . . . , 2.1, 2.2, . . . (unless there are very few theorems).

Use a simple and self-explanatory system for marginal headings. My first prize for what *not* to do goes to von Neumann and Morgenstern [23], in which the following succession is typical [pp. 140-141]:

(16:15)
(16:C)
(16:16:a)
(16:16:b)
16.4.2
(16:D)

By the way, their system is *very* logical.

Number only those expressions you refer to; the numbers then serve as clear ignals. (This is likely to require checking and revising.) In addition, the copy ditor will be free to relocate formulas, as convenient, between unnumbered dislays and the text. (Some *textbook* authors number almost *all* displays, so that tudents can refer to them easily when asking questions.)

5.6 Multiple indices

Simple combinations of indices, such as

$$x_3^2,$$

vhich are indispensable to the mathematician, are easy for the copy editor and he compositor to comprehend, and easy (in context) for the reader to assimilate. nvolved combinations of higher order invite typesetting errors. I recall the deeat of a major research journal at the hands of the standard symbol

$$2^{2^{\aleph_\alpha}},$$

vhich it printed as

$$2^{2\aleph^\alpha};$$

ne author protested, and the journal printed a "correction" [25]:

For $2^{2\aleph^\alpha}$ *read* $2^{2\aleph^\alpha}$

ne way out: exp exp $\aleph_\alpha$ (But see the remarks in Section 8.5.)

A complex symbol, even when printed correctly, may require too much unaveling by the reader; moreover, second-order indices are necessarily small and ften hard to read. Try to replace an elaborate symbol by a series of simpler nes; you may be able along the way to simplify the intricate mathematical ar-

gument that led to it. My candidate for the all-time champion is the following concoction by Sierpinski [24, p. 164]:

$$t_{n_k^{\xi_k^\alpha}}^{\xi_k^\alpha}$$

Actually, it was printed correctly—he must have climbed into the printing press for a last-minute check. The Appendix analyzes Sierpinski's paper and shows how this complexity could have been avoided.

5.7. Suppress useless information

Do not routinely announce all the variables a temporary symbol depends upon. If you are deriving the formula

$$1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}, \qquad r \neq 1,$$

you may wish to call the sum S_n, compute rS_n, subtract, and solve. No. Just call it S.

Quantifiers invite the same temptation. In the assertion

$$\forall \alpha,\ \forall i,\ \exists j,$$

the dependence of j upon i and α is built into the symbol. When we shout at our students that δ depends on ε, it is for emphasis in a bewildering context, not from logical necessity. (If you report at the PTA that every teacher in the school drives a car, you don't add: "depending on the teacher".) Don't go out of your way to make general remarks about all the

$$j_i^\alpha.$$

It may be that you are going to pick an α and an i only once; then on that occasion, you can call the corresponding j just plain j. The Appendix presents an outstanding example.

6. ENGLISH USAGE

Use good English, writing carefully, clearly, and correctly. Your article does ɔt have to qualify as a literary masterpiece, but it should adhere to the accepted inciples of correct diction, grammar, punctuation, and spelling. Among the veral reasons for this are the very practical ones that incorrect English is dis- acting to the serious reader and is likely to be unclear. There are less benign ɔssibilities. Richard Mitchell [8] is a forceful exponent of the thesis that un- ammatical writing and incorrect spelling betray a lack of precise thinking. en Jonson said the same thing several centuries ago [28]:

> *Neither can his Mind be thought to be in Tune, whose words do jarre; nor his reason in frame, whose sentence is preposterous.*

ι fuller quotation is given in [7, p. 3]).

For convenience, I have assembled some do's and don't's, which I present re. Readers interested in pursuing these matters further can choose from nong a variety of authoritative treatises. The bibliography lists several that are ritten with good humor and suitable for light and pleasant browsing (despite ing informative).

6.1. Style

Write simple, unaffected prose. Writing is harder then speaking be- use your tone of voice isn't available to help make your point clear. Keep ntences crisp—think of what you want to say and say it. Mathematics is hard ough to read without convoluted writing that makes it harder.

Use the active voice. All writing experts agree on this. Passive con- uctions leave the reader wondering who is doing what to whom; moreover, ey encourage a verbose style, which makes things worse.

It has been noticed that **X** *We have noticed that* √
Occurrences were observed in which **X** *I saw* √

Avoid ponderous nouns. Instead of long nouns ending in "tion", use verbs:

for the preparation of (?)
for preparing √ *to prepare* √
for the acquisition of (?) *to acquire* √ *to get* √

Sentences that start off with ponderous nouns are doubly insidious, as they encourage a passive construction:

> *Confirmation observation of civilization domination relationships are undergoing reexamination completion.* (?)

Avoid abstract nouns that promise more profoundity than the context provides. The classic is "methodology" [*the methodology of least squares* (?), *Newton's methodology* (?)]. Two current favorites are "objective" [*the objective of* my affections (?)] and "motivation" [*the profit motivation* (?)].

Gilbert & Sullivan's Mikado did *not* announce:

My objective all sublime (?)
I shall achieve in time—
To let the punishment fit the crime—
The punishment fit the crime.

That was his object (as he correctly asserted [27]); his objective was justice. (Fowler would even say the (grander) *object* was justice, reserving "objective" for contexts that do not strain the metaphor with *military objective* [14]).

The newspaper quotes an entrepreneur:

> *My motivation was simple: to make as much money as possible.* **X**

That was his motive; his motivation was greed. (*Motivate* means to provide with a motive.)

Recommendation: Try the simple form first; if it works, keep it.

6.2. Some distinctions in meaning

***Appraise* vs. *apprise*:**

> The editors will *appraise* your manuscript and *apprise* you of their decision.

***Comprise* vs. *compose*.** A set *comprises* (embraces, consists of, is composed of) its elements; the elements *compose* (or constitute) the set. If you are not sure about "comprise", try "embrace" (or "include"); if it's wrong, use "compose".

R comprises *the rationals and the irrationals.* √

R *is comprised* [*embraced*] *of the rationals and the irrationals.* **X**

R is composed of *the rationals and the irrationals.* √

Comprise is all-inclusive: **R** does not comprise the rationals.

***Ensure* vs. *insure*.** Reserve the second for taking out insurance; for *guarantee*, use the first.

Send me a calculator to **ensure** *correct answers; and* **insure** *the package.*

***Inclusion* vs. *containment*.** Reserve the second for military strategy; in mathematical writing, use the first.

If $A \subset B$ *and* $B \subset C$, *then* $A \subset C$: **inclusion** *is transitive.*
The Allies' **containment** *of the enemy forces led to victory.*

***Quickly* vs. *soon*.** I know more than one mathematician who writes, "I hope you will make up your mind quickly". This allows you to put everything off for five months and then make a snap decision, but rules out deciding tomorrow after six hours of careful thought—in short, confuses t with dx/dt.

***Which* vs. *that*.** This distinction might seem a high-flown affair, but it rests on being able to recognize what a definition is, and mathematicians are good at that. "That", present or understood, is the defining pronoun; "which", which invariably follows a comma, is the nondefining one:

Here's the calculus book **that** *I bought yesterday.* √
[Defines which one.]

Here's the calculus book I bought yesterday. √
[Defines which one.]

Here's the calculus book, **which** *I bought yesterday.* √
[Adds information about the one already under discussion.]

False signals. You can't always guess the meaning of a word from its sound:

bemuse: *stupefy*	fulsome: *gross*	noisome: *disgusting*
enormity: *heinousness*	hypothecate: *mortgage*	notorious: *infamous*
fortuitous: *accidental*	meretricious: *tawdry*	officious: *meddlesome*

To be safe, do not use fancy-sounding but slightly unfamiliar words without looking them up.

6.3. A few matters of grammar

As far as. A popular solecism is to drop the verb:

As far as algebra, I like it. **X** **As for** *algebra, I like it.* √
As far as *algebra* **goes [is concerned]**, *I like it.* √

Before long, I suppose, we will be hearing: "As far as that, it's a good idea". (As far as I, it's a bad idea.)

Different than. **X**

A *is different than* B. **X** A *is different* **from** B. √

Apparently, the "er" sound in "differ" suggests a comparative—like the slogan on the delivery truck:

Faster than rail, regular than mail. *(?)*

She explained it to I. **X**

They invited my husband and I to the colloquium. **X**
They invited my husband and **me** *to the colloquium.* √

She explained the proof to Bob and I. **X**
She explained the proof to Bob and **me**. √

It will be good for he and I to discuss it. **X**
It will be good for **him** *and* **me** *to discuss it.* √

The test is easy—leave out the other object:

They invited **me**. √
She explained the proof to **me**. √
It will be good for **him**. *It will be good for* **me**. √

One of those are. **X** A pernicious habit—a plural verb in response to the word it follows, although the actual subject is singular:

The best one of those are in the book. **X**
The best one of those **is** *in the book.* √

The character of the problems have changed a lot. **X**
The character of the problems **has** *changed a lot.* √

The caliber of those people are not that of ours. **X**
The caliber of those people **is** *not that of ours.* √

The test is easy—omit the phrase:

The best one **is** *in the book.* √
The character **has** *changed a lot.* √
The caliber **is** *not that of ours.* √

One of those who. A standard trap:

She is one of those who enjoys mathematics. **X**
She is one of those who **enjoy** *mathematics.* √

The test is easy—invert the word order:

Of those who **enjoy** *mathematics, she is one.* √

6.4. *In terms of*

This phrase appears almost invariably as mere noise, at best displacing a simple *in, to, for, of,* or *by.*

How is he doing in terms of calculus? **X**
How is he doing in calculus? √

How shall we respond in terms of her letter? (Meaning *?*)
How shall we respond **to [in view of** (*?*)] *her letter?* √

That's important in terms of getting information. **X**
That's important **for** *getting information.* √
That's important for information. √

We should respond to their challenge in terms of inspiring the students. **X** (Meaning *?*)
We should respond to their challenge **of** [**by** (*?*)] *inspiring the students.* √

In terms of schedule, let's meet Wednesday. **X**
Let's meet Wednesday. √

Express the roots **in terms of** *the coefficients.* √

6.5. Some distinctions in spelling

He **complemented** *the lecture with slides and his audience* **complimented** *him for it.* √

She was **discreet** *when criticizing the article on* **discrete** *mathematics.* √

Their **loose** *reasoning will* **lose** *the argument; that will be their* **loss.** √

Mathematical induction is the **principal principle** *for proving theorems about the integers.* √

Do your students call it the "communitive" law too? Be sure to hold up your end:

accidently, incidently **X**	accident**al**ly, incident**al**ly √
concensus **X**	con**s**ensus √
correspondance, existance, inadvertant, occurrance **X**	correspond**e**nce, exist**e**nce inadvert**e**nt, occurr**e**nce √
indispensible **X**	indispens**a**ble √
noone **X**	no one √ none √

(The first two are easy: a "ly" adverb has to start from an adjective, not a noun. The third is easy, too: think of *consent.*).

6.6. Some ups and downs of punctuation

Two "comman" pitfalls. Either set off a phrase with commas, or don't set it off; that means enclose it in an even number of commas.

We will learn in the next chapter, how to solve it. **X**
We will learn, in the next chapter, how to solve it. √
We will learn in the next chapter how to solve it. √

Cultivate an affection for the semicolon; it was invented for combining two closely related sentences into one, for better flow. A comma is insufficient:

This shows that $x = 2$, *therefore* $y = 3$. **X**
This shows that $x = 2$; *therefore* $y = 3$. √

EXCEPTIONS:

I came, I saw, I conquered. √ *I think, therefore I am.* √

The dash (—). I use this often in place of a colon or semicolon, because it stands out so well—but indiscriminate use devalues it.

Do not type a hyphen when you intend a dash. Use two hyphens - - or type the underline a half-space up.

Hyphens. I use hyphens to resolve otherwise ludicrous constructions:

A non-PhD granting institution. **X** [Granting the non-PhD?]
A non-PhD-granting institution. √

An ex-college professor. **X** [A professor at an ex-college?]
An ex-college-professor. √

In the latter example, Bernstein [12] prefers to regard "ex" as an actual word and do without either hyphen. All authorities agree that

A former college professor √

is best of all.

The slash (/). The following actual quotes show the slash representing:

and:	*a list of hotels/motels*;
at:	*The University of Texas/Austin*;
in:	*our correspondent Leslie March/New York*;
or:	*study math and/or physics*;
a comma:	*algebra/topology/analysis and probability*;
a hyphen:	*a math/physics major*.

And/or, I suppose, we could define the slash in "and/or" to mean *and/or*.

The principal effect of a symbol with so many interpretations is to betray the writer's cloudiness of thought.

6.7 Latin words

It is good to steer clear of foreign words unless you are sure of your ground.

I.e. vs. ***e.g.*** Don't use "i.e." when you mean, e.g., "e.g.". The first stands for *id est*, = *that is*; the second is *exempli gratia*: "of example for the sake"—i.e., *for the sake of example*.

Curriculum vitae = "course of life". For short, use *vita* ("life"). But "my curriculum vita" **X** and "my vitae" **X** are illiteracies. The plural, by the way, is *curricula vitae* ("courses of life") or *curricula vitarum* ("courses of lives"); or, for short, *vitae* ("lives").

Noon is 12:00 **m**, for *meridies*, = "midday".

Plurals. *Data*, like *curricula*, *extrema*, *maxima*, and *minima*, is plural, as are *bacteria*, *media*, and *symposia*, and, from the Greek, *criteria* (sing. *criterion*) and *phenomena* (sing. *phenomenon*).

This data is interesting. **X** **These** *data* **are** *interesting.* √
That criteria is the one. **X** That **criterion** is the one. √

6.8. Pre-plan ahead for the future in advance

We used to reserve our seats, and packaged food came sealed. Today our seats are *pre*-reserved; and *pre*-packaged food is *pre*-sealed. Remember, then:

Have your students *pre-learn* the material for the test.
Pre-prove your theorems before publishing them.
Pre-prepare your lectures.

In view of (8), then, $\alpha \notin S$.

We have shown that for $\alpha \neq \beta$, if $\alpha \notin A_\beta$ then $\alpha \notin S$; that is, if $\alpha \in S$ then $\alpha \in A_\beta$. Since A_β is countable, S is countable. ♦

Comment on the theorem. The original article uses the language of transfinite ordinals; the hypothesis of the theorem is the *continuum hypothesis* (CH), which states that the uncountable sets in **R**, and the set of all countable ordinals, all have the same cardinal as **R** itself. The hypothesis as stated in this Appendix is an adaptation of (CH) to the particular problem. If we assume (CH), then the ordered set S consisting of the negative integers followed by the countable ordinals can be indexed by **R**, and the sets $A_\alpha = \{x \in S: x < \alpha\}$ are as stated. The converse can also be proved directly, but note that it is immediate from the conclusion of the theorem: every uncountable set in **R** is carried by some f_k onto **R**, hence must have the cardinal of **R**.

Annotated Bibliography

MATHEMATICAL WRITING

Essays

[1] R. P. Boas, *Can We Make Mathematics Intelligible?*, Amer. Math. Monthly **88** (1981), 727-731.
Required reading.

[2] Paul R. Halmos, *How to Write Mathematics*, Enseign. Math. **16** (1970), 123-152; reprinted in Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, and Jean A. Dieudonné, *How to Write Mathematics*, American Mathematical Society, 1973, 19-48, and in P. R. Halmos, *Selecta, Expository Writing*, Springer, 1983, 157-186.
Required reading.

[3] J. E. Littlewood, *A Mathematician's Miscellany*, Methuen, 1953.
A charming classic that every mathematician should read in its entirety in any case. The piece relevant to the present manual is described in Section 3.4.

Author's manuals

[4] American Mathematical Society, *A Manual for Authors of Mathematical Papers*, Eighth Edition, pamphlet, 20 pp. American Mathematical Society, 1984.
A basic manual, containing an excellent short section on organizing the paper (tactfully marked "for beginners") and a great deal of material on technical matters such as styles of notation and the form of abstracts. The manual has been updated at irregular intervals since the first edition in 1962; a precursor with the same title, style, format, and galley-proof example was issued by the Society in 1943 as a supplement to the March issue of the Bulletin.

[5] Harley Flanders, *Manual for* MONTHLY *Authors*, Amer. Math. Monthly **78** (1971), 1-10.
Good advice in no-nonsense style.

[6] Ellen Swanson, *Mathematics into Type*, Revised Edition, paperback, 90 pp. American Mathematical Society, 1979.
An exhaustive work, as suggested by its subtitle: *Copy Editing and Proofreading of Mathematics for Editorial Assistants and Authors.*

THE ENGLISH LANGUAGE

Essays and criticism

[7] Richard Mitchell, *The Leaning Tower of Babel*, 279 pp. Little, Brown, 1984.
Excerpts from *The Underground Grammarian*, Mitchell's hard-as-nails monthly attack on the evils of the education establishment. The book is listed here for the passage by Ben Jonson referred to in the introduction to Chapter 6.

[8] ——————, *Less Than Words Can Say*, 224 pp. Little, Brown, 1979.
The author at his brilliant and acerbic best versus the educationists. Chapters 3, 5, and 6 contain ideas that everyone, even mathematicians, might well ponder.

[9] William Zinsser, *On Writing Well, An informal guide to writing nonfiction*, Harper and Row, 1976.
A lot of good sense served up in a friendly manner.

Manuals of style

[10] Claire Kehrwald Cook, Modern Language Association, *Line by Line, How to improve your own writing*, paperback, 219 pp. Houghton Mifflin, 1985.
A low-key presentation of simple, sensible suggestions and clear explanations. But the publishers get a D- for consistently crooked right-hand text with nonexistent margins.

[11] William Strunk Jr. and E. B. White, *The Elements of Style*, Third Edition, paperback, 92 pp. Macmillan, 1979.
A staple. The short paragraph under the heading *Omit needless words* is a gem.

Dictionaries of English usage

[12] Theodore M. Bernstein, *The Careful Writer, A Modern Guide to English Usage*, paperback, 487 pp. Atheneum, Eleventh Printing, 1986.
Far less ambitious than *Follett* or *Fowler*, and a warmer spirit: Large type, an open page, and an author who is talking to *you* rather than writing an essay.

[13] Wilson Follett, *Modern American Usage*, Edited and completed by Jacques Barzun, Thirteenth printing, paperback, 436 pp. Hill and Wang, 1986.
Presumably an attempt to be the American *Fowler*. I do recommend the piece on possessives and the convincing argument for pronouncing "rationale" as *ray-shun-ay'-lee*. But I am put off by two items in my field of greatest expertise: (i) he confuses *playing by ear* with *playing by heart* (well, that's only off by two letters), and (ii) he likens the proscription of the split infinitive in English to that of parallel fifths in music, whereas the first is mostly whim while the second has a sound basis (sorry about that). Finally, the pages do not lie flat and the inside margins are tiny, so that reading the text near the binding is a nuisance.

[14] H. W. Fowler, *A Dictionary of Modern English Usage,* Second Edition, Revised and Edited by Sir Ernest Gowers, 725 pp. Oxford University Press, 1965.

The peerless classic, revered for the wit and artistry of its writing as much as for its pungent advice. Must be sampled slowly; ideal for browsing: pick out any page and enjoy the language. Be sure to read the section on the "fused participle", especially the first column (and, for fairness, the last), as well as the discussion of the split infinitive.

Dictionaries of synonyms

[15] J. I. Rodale, *The Synonym Finder*, 1359 pp. The Rodale Press.

Valuable for suggesting possibilities or supplying the word that was at the tip of your tongue. Arranged alphabetically.

[16] *Roget's International Thesaurus*, Fourth edition, paperback, 1317 pp. Harper and Row, 1977.

The classic reference for synonyms; immensely valuable. The material is arranged by categories of meaning, so that closely related ideas appear near one another. An alphabetical index of 500 pages, with entries subclassified by shades of meaning, gets you started.

[17] *Webster's New Dictionary of Synonyms*, 907 pp. Merriam Webster, 1984.

Arranged alphabetically. Commentaries and quotations elucidate the distinctions among synonyms; these features are absent from Rodale and *Roget*. There are also lists of analogous words, contrasting words, and antonyms; *Roget* has an equivalent feature, but Rodale has nothing. The coverage seems skimpy. *Examples.* (1) "Laud" is here, but neither "laudable" nor "laudatory", while the other two works give all three; (2) between "protect" and "proud", *Webster* has 6 entries; Rodale, 12; *Roget*, 35.

English dictionaries

[18] *The American Heritage Dictionary of the English Language*, First Edition, Unabridged, 1550 pp. Houghton Mifflin, 1969.

In a class by itself; reputed to have been written as the answer to Webster's permissive Third International. The usage notes are particularly valuable. But I understand that later editions have been watered down.

[19] *The Random House Dictionary of the English Language*, 2059 pp. Random House, 1983.

More comprehensive than *Heritage.* Includes helpful dictionaries to and from French, Spanish, Italian, and German. Exactly twenty years ago this month, I wrote the publisher that the interval [heimisch, Heimsuchung] should *precede* [heimtückisch, heissen] (taking the occasion to allude to the title of the work); the mislocation has withstood all intervening printings.

MISCELLANEOUS REFERENCES

[20] Edgar Asplund and Lutz Bungart, *A First Course in Integration*, Holt, Rinehart and Winston, 1966

[21] P. Erdős, *Some Remarks on Set Theory*, Proc. Amer. Math. Soc. **1** (1950), 127-141.

[22] Lester R. Ford, Sr. and Lester R. Ford, Jr., *Calculus*, McGraw-Hill, 1963.

[23] John von Neumann and Oskar Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, 1947.

[24] W. Sierpinski, *Sur une Certaine Suite Infinie de Fonctions d'une Variable Réelle*, Fund. Math. **XX** (1933), 163-165; **XXIV** (1935), 321-323.

[25] Trans. Amer. Math. Soc. **64** (1948), 596.

[26] Lewis Carroll, *Alice's Adventures in Wonderland*, Chapter XII, *Alice's Evidence.*

[27] W. S. Gilbert and Arthur S. Sullivan, *The Mikado.*

[28] Ben Jonson, *Explorata—Timber, or Discoveries Made upon Men and Matters.*