Timeline of Marie-Sophie Germain

The story of a truly remarkable woman

Marie-Sophie Germain born **April, 1 1776**

Germain recieves Lagrange's lecture notes
Germain turns in paper under psudoname LeBlanc
Legrrange insists on meeting author of said paper, Germain's true identity is exposed

1804 - 1806 Germain (as M LeBlanc) and Gauss interact often through letters.
1804 -Sophie Germain solves Fermat's Last Theorem for n=p-1 for prime p=7mod8 and sends to Gauss

1775 | 1776 | 1789 | 1794 | 1801 | 1805

American Revolution 1775-1783

French Revolution begins

L'Ecole Polytechnoque opens in Paris

Gauss publishes *Disquistiones Arithmeticae*

1806 Germain asks French general to check on Gauss
Germain's identity exposed to Gauss

1809 - Germain attempts at *Prix Extraordinaire (PE)*

1811 - Germain enters *PE* but is rejected due to a flaw

1808 - last letter from Gauss

1813 -Germain reenters *PE* awarded Honorable mention

1825 - Legendre prints Sophie Germain's proof in his *Theorie des Nombres*

June 27th Germain dies of Breast Cancer

1806 | 1810 | 1815 | 1831

1806 - Napolean's army in conflict with Prussia

1808 - Chladni visits paris to demonstrate modes of vibration
1809 - Chladni's patterns become subject of *Prix Extraordinaire*

1811 - *Prix Extraordinaire* reinstated, due 1813

1813 - *Prix Extraordinaire* reinstated, due 1815

Sophie Germain:

The struggles and accomplishments of a remarkable woman in mathematics

In the age of revolution, just as the American colonists struggled for independence from England, Marie-Sophie Germain was born into a wealthy family in Paris, France. Her father, a merchant, and later a director of the Bank of France, had no way of knowing that his child would one day become such an outstanding mathematician. Germain began her love affair with mathematics at the age of 13, as the French Revolution, occurring just outside her front door, confined her to her home. While her parents were the first to discourage to her study of math, they were not the only obstacles. Germain's preferred strategy to avoid these obstacles was to communicate with other mathematicians using a pseudonym. She felt it was the only way the mathematics community would take her seriously, but her true identity was always eventually exposed. She succeeded in gaining the confidence of many prominent mathematicians of her time and made accomplishments in the fields of number theory and physics.

Young Sophie

Marie-Sophie Germain was born, the second of three daughters, to parents Ambroise-François Germain and Marie-Madeline Gruguelin. By the age of 13, her personality emerged to be a bit shy, awkward and timid. Outside her door, in the streets of Paris, the French Revolution was taking place. When the family would have visitors, the house would be full of talk about politics and change. Sophie recoiled from the turmoil of the street, and boredom of long days in the house, by spending a lot of time in

her father's vast library. Perhaps we could credit Archimedes[3] for bringing out the talents of Sophie Germain for the rest of the world to enjoy. It is said that when reading Montucla's *Histoire des Mathematiques* that Germain came across the story of Archimedes's death, slain in Syracuse by a Roman soldier. Archimedes was engrossed in studying a figure he had drawn in the sand, and he did not respond to the soldier as he approached. Frustrated and ignorant of who this great mathematician was, the soldier killed Archimedes on the spot. This struck Germain as heroic and she figured, if a man could be so interested in a problem, as to be killed for it, then the subject must be very intriguing. It was then that young Sophie decided that she, like Archimedes, would dedicate her life to being a Geometer.[1] She began this journey with Bezout's standard text in mathematics, a book that, at the time, could be found on most intellectuals' bookshelves.

Germain was relentless in her studies of mathematics. At first, her parents were not very accepting of how serious their Sophie was about the subject. While it was expected that women of the time were to know enough of science and mathematics to carry on a polite conversation at a party, her parents found it difficult to believe that she would be able to find any real use of the subject. They were also concerned about Sophie's health and thought she was spending too much time with math, and not enough with sleep! In fact, they took away her light and her clothing at nighttime in order to discourage her from studying through the night. But of course, living up to the revolutionary spirit of the times, Sophie refused to be hindered, she waited until the house was asleep, and found ways to study through the night, even in such cold that the ink

[1] In her time the term "geometer" was understood to mean a mathematician in general, rather than the specialized mathematician who studies geometry.

would freeze! One morning, Sophie's parents found her asleep over her notes, bundled in blankets. They realized that their daughter could not be stopped, and agreed to allow her to study during the day.

Germain as "Leblanc"

In 1795, Germain was at a ripe age to take her studies to the universities and academies that were around at the time. In this year, *L'Ecole Polytechnique* was established in Paris, but unfortunately for Germain, the academy did not accept women as students. However, it was common practice of the school to make lecture notes available to the public. Germain obtained lecture notes from a chemistry course and Lagrange's mathematics course. Of course she was at a disadvantage of the actual student's of Polythechnique, who could casually converse with each other about mathematics. The school also had a policy that students could submit papers to the professors at the end of the term. Germain submitted a paper of her observations to Lagrange under the false name M. Antoine Leblanc. Her hidden identity did not last long. Lagrange read her paper he was impressed with her originality and insight, and insisted on meeting the author. When Germain's identity was discovered, Lagrange was still impressed, and agreed to help her in her studies. He introduced her to areas of mathematics such as number theory in which she read Fermat, Legendre and Gauss. This encouragement was important to Germain. Soon word got around Paris and she was approached by many mathematicians who offered to help her in her studies. However, since none of them could provide the advanced level of instruction she wanted, she continued to work on her own.

One of the problems that Germain worked on was Fermat's Last Theorem. Fermat had written in the margins of a book that he had a proof for the fact that $x^n + y^n = z^n$ has no

solution for integers n>2. However, no such proof has ever been found. Mathematicians have been trying to achieve a proof ever since. Mathematicians had been able to proof the statement for certain values, but since there are infinite values for n, looking at them independently was not going to proof the statement. Germain made much progress in this field. She found a proof for the case where $n = p - 1$ for some prime $p \equiv 7 \bmod 8$. Germain sent these results to the new up and coming mathematician of the time, Friedrich Gauss. She had read his *Disquisitiones Arithmeticae* and admired his work. Of course, insecure that he may not take a woman seriously, she wrote to him under her familiar pseudonym Antoine LeBlanc.

Gauss and Germain continued regular correspondence through letters from 1804 – 1806. Germain would send her results to some of the problems discussed in *Disquisitiones Arithmeticae* and he would always respond, though not always in a timely manner, with encouraging letters. Gauss was flattered by LeBlanc's interest in his work and impressed by 'his' abilities.

In 1806, napolean's armies were preparing to invade and occupy Prussia. Germain, remembering the fate of her fisrt inspiration, Archamides, was concerned for Gauss's safety. She asked a friend of the family, French General Pernety to find Gauss and be sure he was safe, and not mistreated. Gauss welcomed the generous deed but admitted he did not know who this mysterious protector, Sohie Germain, was. In order to clear things up, Germain sent him a letter explaining her identity. Gauss, like Lagrange, responded as a great man should, and was not turned off by his pen pal being a woman. In fact he was even more impressed by her talents, that

> "When a woman, because of her sex, our customs and prejudices, encounters infinitely more obstacles than men in familiarizing herself with their knotty problems, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent, and superior genius."[2]

Sophie Germain's correspondence with Gauss is impressive. That she could attract and maintain the attention and respect of Gauss, who is believed to be the greatest mathematician of his time, shows that she was a very talented woman. His response to her true identity confirms that he is a great man, without sexist prejudices. In fact, as her reputation spread throughout the mathematics community, she no longer needed to hide her identity. Men knew she was capable and were willing to consider her talents. But it was not so much her accomplishments, than what she was unable to accomplish that shines the light on the disadvantages of being a woman trying to make a name for herself in the mathematics community of the time.

Prix Extraordinaire

In 1808 a German physicist by the name of Ernst Chladni came to Paris to demonstrate his observations patters in the vibrations of elastic surfaces. In 1809, Napoleon made Chladni's patterns the topic of the *Prix Extraordinaire,* a contest in physical science put on by the Paris Academy of Sciences, First Class of the Institute. The prize was to be 3000 francs. The judges of the competition would be Legendre, Laplace, Lagrange, Lacroix, and Malus. Legendre and Lagrange had been mentors to Germain in the past. The task to be accomplished was,

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[2] Letter from Gauss dated 30, April, 1807 as reprinted in Bucciarelli, pg. 25.

"The class has thus proposed, for the subject of the prize, the development of a mathematical theory of the vibration of elastic surfaces, and a comparison of this theory with experiments"[3]

Sophie Germain took on this challenge. It was here that her lack of a structured education left her at a true disadvantage. Many of the fundamentals and methods that most mathematicians take for granted, Germain never properly learned. The foundations from which she started in this project were Lagrange's *Mècanique* and Euler's memoir on the vibrations of elastic beams. Germain's received help from Legendre at first, but he discontinued his assistance before her entry was complete. At the deadline, October 1, 1811, Sophie Germain's entry was the only one the academy received. However, she was not awarded the prize because her entry had a serious flaw in it. Germain had problems with her double integrals.

In 1811, because the prize was not awarded, the contest was extended. The new deadline would be October 1, 1813. In her first entry, Germain assumed the hypothesis that at any point, the elasticity can be represented by (1/r) + (1/r`). However, because of her difficulty with double intervals she did not derive the correct equation. Lagrange took her hypothesis and correctly derived the equation

$$\frac{d^2z}{dt^2} = k^2\left(\frac{d^4z}{dx^4} + \frac{2d^4z}{dx^2dy^2} + \frac{d^4z}{dy^4}\right)$$

Germain accepted this equation to be true and in her second attempt at the prize, she focused on trying to derive this equation. She continued to have difficulty deriving the equation because she did not have the fundamental understanding needed to fully understand Lagrange's *Mècanique*, which was needed to complete the derivation. Despite

[3] The announcemtn of this contest was published in Chladni's book, *Traite d'acoustique.* My refernce has been taken from Bucciarelli, pg. 35.

gaps in her skills, Germain was more confident in her capabilities and she was sure she was headed in the right direction. She worked on this memoir is isolation. She did not seek help from her peers this time. As she turned in her second entry to the contest, she was more confident in her mathematics, but less confident that the judges would be objective and give her memoir the attention she felt it needed and deserved.

Once again, Germain's memoir was the only entry into the contest. Since she knew what her goal was, the equation produced by Lagrange, she derived the correct equation. However she failed to present proper justifications for her hypothesis. Her work was so full of errors in what she chose to include and omit, that she was unable to convince the judges. However, though her analysis was flawed, parts of her memoir were correct. Her explanation of the phenomenon was worthy of an honorable mention, which the committee awarded her.

Once again, because the contest had no winner, it was extended for another 2 years. The new deadline would be October 1, 1815. This time, Germain stuck to her previous approach and worked toward correcting expanding on her previous memoirs. Finally, in 1815, she was awarded the prize but "with reservations." Her memoir was still not completely convincing and the judges criticized her for "the way she moved from her hypothesis to its justification, i.e., for her mathematics."[4]

Although Germain did not completely solve the problem, her difficulty in finding a convincing solution was rooted in her lack of thorough, organized training as she developed as a mathematician. However, her memoirs were undoubtedly valuable in the continued progress in the field, and the emergence of a plate theory and a theory of elasticity. While these theories were found by building on ideas in Germain's memoirs,

[4] Bucciarelli, pg. 35.

she did not receive due credit in the placard at the Eifel Tower that names prominent scientists and mathematicians who contributed to the elasticity theory.

Regardless of the shortcoming of her memoirs, the award of this prize was Germain's ticket into the scientific community in Paris. Thanks to Fourier, who helped Germain in her further development of her memoirs, she was the first woman, other than the wife of a member to attend lectures at the academy. She also now felt confident that she was a part of this community. However, she continued to have trouble getting worthy responses from her peers. She would send memoirs to different mathematicians in the community, they would reply politely, but because of her stubborn personality and lack of fundamental skills, they avoided her rather than giving her constructive feedback and helping her correct her errors. They were condescending rather than helpful.

> "The elasticity problem had now moved into a community to which she did not and could not belong, even though she did not know it, and even though quite independently of her mathematical inadequacies, though these inadequacies came from the same source, namely her being a woman and thus excluded from formal education. That she had the native ability and intuition is illustrated by the remarkable amount that she taught herself and by the flashes of insight that appear in her work."[5]

Throughout her attempts at the *Prix Extraordinaire,* and also afterwards, Germain struggled with the mathematics community who made her out to be somewhat of a fool by allowing her to press on in the field of elasticity without giving her constructive feedback on her errors and not leading her in a more appropriate direction.

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[5] Louis L. Bucciarelli,. *Sophie Germain: An Essay in the History of the Theory of Elasticity*. (Dordrecht, Holland: D. Reidel Publishing Company, 1980), 108

Back to Number Theory

Once Germain received the award for the *Prix Extraordinaire*, she went back to the mathematics she had worked on previously, which she have a very natural ability for; that is, number theory. Earlier, she had shown Fermat's Last Theorem was true for the case, n=5. Now, she worked to generalize this proof to show that it is true of an odd prime n if there is a p such that

(a) $x^n + y^n + z^n \equiv 0 \bmod(p)$ implies $xyz \equiv 0 \bmod(p)$ and

(b) n is not an n-th power $\bmod(p)$ [6]

This is called *Sophie Germain's Theorem.* It proves case 1 of Fermat's Last Theorem if $2n+1$ is prime (p) all nth powers are congruent to 1, -1, or $0\bmod(p)$, these primes are called *Sophie Germain Primes*.

This work is perhaps what Sophie Germain is most recognized for. A self taught woman, who was never fully accepted into the mathematics community, made significant progress in the fields of elasticity and number theory. One could only imagine the impact she may have had on mathematics if she was allowed formal education and full access to the institutions privy to men in her field. Sophie Germain died on June 27, 1831 of breast cancer without ever meeting Gauss face to face, and before receiving the honorary doctorate degree that Gauss had convinced the University at Gottingen to award her. Upon her death, Libri, whom had befriended Germain in her later year, wrote a flattering obituary, which is referenced often in writings about the story of Sophie Germain.

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[6] Hall, Natascha, Jones, Mary and Jones, Gareth. "The life and work of Sophie Germain." *Mathematische Semesterberichte.* 15, (2004): 1-6.

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