

**MATH 800 FOUNDATIONS OF MATHEMATICS  
EPILOG**

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**Measured student outcomes**

Here are the measurable student outcomes listed on the course syllabus, tabulated with actual measured outcomes on a corresponding assignment. (Some assignments, for which there was no clear correspondence, aren't mentioned here.) Each outcome is the sum  $s$  of the scores 0, 1, 2 = poor, ok, fine for the  $n = 9$  students who completed the course. A final column rates the class's performance on each assignment as poor, ok, or fine, according to whether  $0 \leq s < 6 = 2n/3$ ,  $6 \leq s < 12$ , or  $12 \leq s$ .

#	Measurable outcome	Assignment	s	rating
1	Manipulate sets and relations algebraically according to ZF (or some other specified) set theory.	Basic Set Th. S3	11	ok
2	Avoid use of extensions of predicates unjustified by ZF.	not assigned		
3	Use ZF to construct functions for comparing cardinalities of sets.	Ax. of Choice, 2 of S3, S4, S6	10	ok
4	Recognize applications of the axiom of choice in algebra and analysis.	Ax. of Choice, 2 of R3-5	12	fine
5	Explain the relationship between various equivalent forms of the axiom.	contained in #4		
6	Apply the axiom in the form of a maximal principle.	Max. Pr., 2 of R3-5	9	ok
7	Manipulate cardinal numbers algebraically according to ZF plus the choice axiom.	Cards II,3 substantial problems	5	poor
8	Distinguish sets that are finite, countable, equinumerous with the continuum, or larger.	Cards II, 2 of R1, R5, R6	5	poor
9	Analyze the Boolean structure of English sentences and arguments.	Tarski 2.12-13, 15	16	fine
10	Rewrite a sentence using no Boolean connectives save 'and', 'or', and 'not'.	not assigned		
11	Rewrite a sentence using no Boolean connectives except 'nor'.	Boolean 6	14	fine
12	Convert a quantifier-free sentence to disjunctive normal form.	Boolean 5	14	fine
13	Determine under what conditions a quantifier-free sentence is true.	Tarski 2.5b	18	fine
14	Explain the relationship between truth of a quantifier-free sentence and its deducibility from an adequate set of Boolean axioms.	not assigned		
15	Analyze the use of quantifiers in English sentences and arguments.	Tarski 1.6	17	fine
16	Classify the theories studied in upper-division courses as first- or second-order.	only partly covered		
17	Formulate those theories in first- or second-order languages.	not assigned		
18	Explain the relationship between the truth of a first-order sentence with no free variables and its deducibility from an adequate set of first-order axioms.	not assigned		
19	Describe the relationships between the axiom of choice, cardinal arithmetic, and the equivalence of truth and deducibility.	not assigned		
20	Describe the Gödel and Tarski limitations on the strength of first-order theories.	not assigned		
21	Discover information about a limited area of foundations study.	term paper	11	ok
22	Brainstorm, outline, document, and write a term paper about it.	term paper	11	ok
23	Present a brief oral report.	in last meetings	13	fine

This course covered some undergraduate material at unusually high speed, and some graduate material. All of the measureable outcomes on which the class rated “fine”, except for #4, concerned undergraduate material. All of the measureable outcomes on which the class rated “poor” concerned graduate material.

Most measurable outcomes not covered by assignments were covered later in the course in lectures, while students were working on term papers and previously assigned homework.

### Term paper grades

I graded the papers using the following scheme, adapted from last year's Math. 300 history course. The borderline between the first two categories is vague. The presence of both was intended to reflect the fact that a student could write a paper worth 5/5 in scope, but covering subjects inappropriate for that course. For the present course, that danger didn't loom so near, and thus it would be better to amalgamate those two categories.

		<i>Paper scores t</i>	<i>Students (9)</i>
Appropriateness:	5	$15.2 \leq t \leq 19.0$	A *****
Scope:	5	$12.4 \leq t < 15.2$	B **
Originality:	4	$9.5 \leq t < 12.4$	C **
Presentation:	5	$7.6 \leq t < 9.5$	D
Total t :	19	$t < 7.6$	F

Paper titles:

- Intuitionism and Constructive Math*
- The Comparison of Two Methods for Constructing the Real Numbers*
- Equivalence of Turing Computable Functions and General Recursive Functions*
- A Survey of Thomsen's Treatment of Elementary Geometry by a Group Calculus*
- Measurement Theory: Development of a Structure*
- Forcing and the Continuum Hypothesis*
- Cantor and Russell*
- Quine's New Foundations*
- Domain Theory as a Basis for Denotational Semantics*

### Homework grades

Assigned homework totaled 63 points.

<i>Homework scores h</i>	<i>Students (9)</i>
$50.4 \leq h \leq 63.0$	A ***
$41.0 \leq h < 50.4$	B ***
$31.5 \leq h < 41.0$	C **
$25.2 \leq h < 31.5$	D
$h < 25.2$	F *

### Course grades

The course grade was based 55% on homework, 10% on class presentations, and 35% on the term paper. All but one student received the full 10% credit for presentations. I did use +/- grades, but chose not to report them here. The highest course score was 94.9%.

<i>Course scores g</i>	<i>Students (9)</i>
$80 \leq g \leq 100$	A ***
$65 \leq g < 80$	B ****
$50 \leq g < 65$	C **
$40 \leq g < 50$	D
$g < 40$	F