MATH 800 FOUNDATIONS OF MATHEMATICS **JT SMITH**
SCHEDULE #15200 **SPRING 2008**

Classroom	TH211	**Office**	MF 12:10–13:00
Class hours	MWF 09:10–10:00	**Hours**	W 14:10–15:00
Office	TH942		
E:mail	smith@math.sfsu.edu	**Phone**	415-338-1623
Internet	math.sfsu.edu/smith		no messages!

Prerequisites To enter this course you should have acquired facility in informal reasoning using logical principles, functions, mathematical induction, the real and complex number systems, limits, and algebraic structures. Those topics are currently part of the content of Math 301, 325, 330, 335, and 370. Some other upper-division or graduate algebra course would be very helpful, since the reasoning in Math 800 is essentially algebraic.

If you're adequately prepared for this course and wish to succeed, you should plan to spend at least eight hours a week on study and writing for this class.

Materials, computing The course texts are *Set Theory and Logic* by Robert R. Stoll, Dover Publications, [1963] 1979, originally published by W. H. Freeman; and *Introduction to Logic and to the Methodology of the Deductive Sciences*, second, revised edition, by Alfred Tarski, Dover Publications, [1946] 1995, originally published by Oxford University Press. Additional course information, including required and supplementary reading and a version of my notes, will be posted on my website. Follow the link displayed above. I'll ask for your email address, and use it to send important messages. That's how I correct errors in lecture and on my website. To prepare the required term paper, you'll probably need to use additional Internet sources, and a good word processor with a formula editor. (I use *WordPerfect* with *MathType*. Most University computers feature Microsoft *Word*. *MathType* is free for our students.)

Overview The "measurable student outcomes," phrased in bureaucratic language for accreditation, are listed at the end of this document.

The foundations of mathematics discipline is devoted to refinement and application of methods originally developed to investigate logical and philosophical underpinnings for mathematics. This course will emphasize arguments connected with the axiom of choice, cardinal arithmetic, and Gödel's and Tarski's famous completeness, incompleteness, and undefinability results about first-order logic, and their applications.

Because SFSU has not regularly offered an undergraduate foundations course, Math 800 will include highly focused but sophisticated treatments of the set theory and Boolean logic required for its goals. You've already encountered some of those topics haphazardly in various other courses. Part 1 of Math 800 will pull them together, then consider the axiom of choice and cardinal arithmetic in depth. I've begun already by posting notes entitled Basic Set Theory. Those are multipurpose, meant for use in other courses. They'll be discussed only lightly in class, in response to your inquiries. The "trivial questions" included there constitute the first assignment, due at the beginning of the third meeting. The rest of part 1 features substantial homework assignments involving techniques covered in lectures. I'll criticize your partial solutions and return them for further work until they're complete or the semester ends. Parts 2 and 3 will also include homework assignments, but of a more routine nature, handled the same way.

During part 1, I'll assign extensive reading on elementary logic from the two course texts. That will be discussed lightly in class, in response to your inquiries, and taken up in depth in part 2. That part culminates with Gödel's completeness theorem, which essentially shows that logic as formalized here is in fact adequate for general use in mathematics. The general and most useful form of the completeness theorem presented here is closely related to the axiom of choice and cardinal arithmetic. That is why part 1 comes first.

Part 3 is devoted to the logical arguments underlying Tarski's undefinability and Gödel's incompleteness theorems. They show that a formalized theory capable of supporting detailed syntactic analysis cannot provide a definition of the class of its true statements, must include statements that are true but unprovable, and cannot prove its own consistency. These will be presented in the context of formal set theory. Gödel's and Tarski's results were presented originally for the formal arithmetic of natural numbers. That version will not be covered in class because it would take too long to convince you that formal arithmetic is adequate for syntactic analysis.

Grading (tentative) Your grade will be based on

- Homework. 55%
- Term paper. 35%
- Class presentations. 10%

You'll be asked occasionally to provide informal in-class discussions of simpler aspects of the course material. In the term paper, you'll investigate some particular topic in foundations, probably including its origin, some aspects of its development, and some example applications. Throughout part 1 of the course I'll suggest possible topics, and discuss them with you individually. By *21 March* we must agree on a topic that matches the course, your background, and your interest. The paper is due *14 May*. You are required to present a brief live report on your topic to the class. Some reports will occur during normal class meetings; others, probably during the "final exam" period, *21 May, 08:00–10:30*.

Pointers Please inform me if you have a disability that requires reasonable accommodation. The Disability Programs and Resource Center (Student Services

Building, Room 110, 415- 338-1041, drc@sfsu.edu), is available to facilitate that.

Reasonable accommodations will be made when observation of religious holidays requires you to be absent from course activities. Please inform me about that well in advance.

When you have difficulty with the course, you should consult me during office hours. If you want to visit me, but can't then, I'll find another time to meet with you. I'll respond to most email queries, but please don't use that medium individually to ask me to repeat complex material that I have presented to the entire class at once. I intend to post on my website detailed outlines of all class meetings. Warning: my phone, 415-338-1623, is useful only when I'm physically in my office, TH942. *Don't* leave messages!

The Mathematics Computing Facility, TH404, will be open regularly for your use. Much useful software is installed on its PCs. Its operating hours will be posted as soon as they're arranged.

The course website, http://math.sfsu.edu/smith/Math800/General/Math800.htm, is already started. It will grow during the semester. I generally post material tentatively some time before it's covered in class, then revise it afterward accordingly. Some parts of the website, particularly its bibliography, will be updated frequently.

Mathematicians expect to receive credit when other scholars use our work, and in return we give credit to others whose work we use. We are extremely sensitive to and intolerant of plagiarism. For a discussion of plagiarism, consult the website

http://online.sfsu.edu/~rone/StudentHelp/Plagiarism.html.

In a case of academic plagiarism, no credit will be given for the assignment in question. I expect your term papers and homework solutions to represent your own work; where you rely on that of others I expect you to give appropriate credit. I will help you learn to do that.

February 20 is the deadline for *adding* this course to your program, or *dropping* it, so that it will not appear on your record. If you wish CR/NCR grading—not allowed in Mathematics major programs—you should discuss that with your adviser and must request it via the Internet by 19 March. All course *withdrawals* must be approved by me and the Mathematics Department Chair; withdrawal is recorded on your record. Withdrawal approval is *not* given after 28 April, except for withdrawal from the University, unless it's justified by events after that date. The College of Science and Engineering enforces such rules much more strictly than other parts of the University.

The Incomplete grade (I) may be assigned only to a student doing satisfactory work until the last few weeks of a course, when events beyond his or her control prevent its completion. If that happens to you, discuss this possibility with me.

Students whose electronic devices interrupt a class will be asked to leave.

"Measurable student outcomes," phrased for accreditation review

1. *Manipulate* sets and relations algebraically according to ZF (or some other specified) set theory;
2. *Avoid* use of extensions of predicates unjustified by ZF;
3. *Use* ZF to construct functions for comparing cardinalities of sets;
4. *Recognize* applications of the axiom of choice in algebra and analysis;
5. *Explain* the relationship between various equivalent forms of the axiom;
6. *Apply* the axiom in the form of a maximal principle;
7. *Manipulate* cardinal numbers algebraically according to ZF plus the choice axiom;
8. *Distinguish* sets that are finite, countable, equinumerous with the continuum, or larger;
9. *Analyze* the Boolean structure of English sentences and arguments;
10. *Rewrite* a sentence using no Boolean connectives save 'and', 'or', and 'not';
11. *Rewrite* a sentence using no Boolean connectives except 'nor';
12. *Convert* a quantifier-free sentence to disjunctive normal form;
13. *Determine* under what conditions a quantifier-free sentence is true;
14. *Explain* the relationship between truth of a quantifier-free sentence and its deducibility from an adequate set of Boolean axioms;
15. *Analyze* the use of quantifiers in English sentences and arguments;
16. *Classify* the theories studied in upper-division courses as first- or second-order;
17. *Formulate* those theories in first- or second-order languages;
18. *Explain* the relationship between the truth of a first-order sentence with no free variables and its deducibility from an adequate set of first-order axioms;
19. *Describe* the relationships between the axiom of choice, cardinal arithmetic, and the equivalence of truth and deducibility;
20. *Describe* the Gödel and Tarski limitations on the strength of first-order theories;
21. *Discover* information about a limited area of foundations study; *brainstorm*, *outline*, *document*, and *write* a term paper about it; and *present* a brief oral report.