1. Ms Kardon reported on her first term paper, about the history of (mostly American) arithmetic texts. This is a difficult area to research, because there has been little interest in the subject over the years, even in schools of education. It has always been hard to find examples in libraries, but I have found many examples for sale cheaply in antique stores. Some are just now becoming available online, due to the recent, somewhat controversial, agreement between Google and various university libraries. Ms Kardon stressed a number of qualities of these texts: their many editions, their inclusion of only a few illustrations, their emphasis on practical problems connected with commerce, for example. In the early 1800s a single text would serve all grade levels, but by the mid 1800s different editions would target narrower age ranges. Since education in those days was available only to a few, the higher-level texts tended to be surprisingly comprehensive. I got bored with school in the eighth grade and decided to study in addition the texts I found in my grandfather's closet. I still have his *Ray's New Higher Arithmetic*, published in 1880, and just found it online via Google Books and Harvard. It's charming and powerful, and contains much lore only hinted at nowadays, in advanced university courses!
2. *Bolzano, Weierstrass, Cantor*
   a. In lectures 26 and 27 I tried to draw a single thread connecting the work of Cauchy, Riemann, Weierstrass, and Dedekind on the foundations of calculus, which leads to that of Peano.
      i. Cauchy provided our familiar formulations in terms of limits, and the mean-value theorem of differential calculus, among other things.
      ii. Riemann provided the definition of integral that we now use in elementary classes.
      iii. Weierstrass provided the ε, δ methods used in our more advanced classes.
      iv. Dedekind and Weierstrass each provided definitions of the concept of real number adequate to support proofs of the basic theorems about continuous functions that are required to establish calculus. I noted that Weierstrass's approach is no longer familiar.
   b. Time prevents my mentioning in class two others in this regard, as follows.
   c. The first serious attention given to some of those basic theorems was by the Czech mathematician B. Bolzano. His story is complicated; I'm not familiar enough with it to give any details. He fell out of grace with the Church, and was forced to live in isolation from academia. His work went unappreciated until the late 1800s, but then did influence others' formulations of calculus.
   d. Georg Cantor, a student of Weierstrass, worked originally on the problem of determining when a function's Fourier series converges to the function value. This led him to deep studies of the real-number system. In an 1872 paper on Fourier series, he published a definition of the concept of real number, alternative to Dedekind's. This was simultaneous with Dedekind's publication, but

many years after Dedekind did his work. The two approaches are very different, but quite equivalent. Both are used in courses today, and have been adapted to yield results in other areas of mathematics. Cantor and Dedekind were in fact friends. Cantor's work in this area led him to develop much stronger tools in set theory than others had used previously. Our entire apparatus of infinite cardinal and ordinal numbers is due to him. He developed set theory over the period 1870–1900, but encountered major resistance to his ideas, and obstacles to publication. In part to circumvent these, he helped found the German Mathematical Society, to which I belong. And he was instrumental in organizing the first International Congress of Mathematicians in Zürich in 1897. Cantor lived until 1918, but spent his last years handicapped by mental illness.

3. *Struik, section 8.26: dissemination to Italy*
   a. Struik mentioned a delegation of young Italian mathematicians who visited the major centers of German mathematics in 1858–1859: Francesco Brioschi, Felice Casorati, Enrico Betti. They brought so much lore back to Italy that their visit is credited with spurring the emergence of Italian mathematics on the world scene.
   b. Brioschi founded that year the *Annali di matematica pura ed applicata*, whose name imitates its French and German counterparts. He returned to Milan to found the technical institute there.
   c. Another journal founded about that time, *Giornale di matematiche*, contained in 1880 a 76-page set of notes on Weierstrass' lectures on analysis, taken in Berlin in 1878 by Salvatore Pincherle on a postdoctoral visit to Berlin. I do not know the story of its founding.
4. *Peano biography, chapter 3*
   a. We've seen that during 1880–1881 Peano served as assistant to D'Ovidio in analytic geometry, etc. He worked 9 hours per week in classes, while D'Ovidio served as rector. And D'Ovidio presented Peano's first papers to the Academy of Sciences in Turin.
   b. In 1881–1882 Peano started as assistant to Genocchi, following his students into calculus. On Genocchi's illness and injury, Peano took over the teaching load.
   c. Peano found the first of his long sequence of errors in standard presentations: a famous result about existence of definite integrals. In 1882 he published a result correcting it: the first of his many papers on fundamentals of analysis.
   d. Peano was working out Genocchi's lectures into book form, which was published Fall 1884. It incorporated that correction, and others.
      i. Genocchi insisted it was *all* Peano's work.
      ii. New wrinkles:
         (1) mean-value theorem (MVT) extension,
         (2) multivariate uniform continuity,
         (3) principles of existence and differentiability of implicit functions,

(4) examples of functions of two variables continuous as you approach the origin in any given direction, but nevertheless not continuous there,

(5) example of noncommutativity of partial derivatives.

I showed the previous two examples in the original: they are exercises in the book. Click here for them. More:

(6) multivariable Taylor series theory,

(7) an analytic expression for the characteristic function of $\mathbf{Q} \subseteq \mathbf{R}$,

(8) improvement of Riemann's integrability definition, using least-upper and greatest-lower bounds. (I'd say that this improvement permitted us to analyze better why certain functions are *not* integrable.)

iii. Note Kennedy's todo about proving the MVT without continuity of the derivative. That's important because of the use of the MVT in estimating errors in numerical approximation of functions.

e. In December 1884 Peano habilitated: he was then qualified to become a professor.

f. Kennedy mentioned student riots in 1885 about Italian colonialism. Shades of 1970 SFSU! Italy would be colonizing various corners of Africa during the next decades, and this was the subject of heated political discussion.

g. In 1886 Peano became professor (instructor) at the nearby Military Academy. That is, he took a second job.

5. *Kennedy, Chapter 4*.

a. Show on pages 7–9 of the Peano 2002 index how Peano's research switched from algebraic geometry (under D'Ovidio's influence) to questions about calculus and its applications.

i. In his `1885a` paper Peano published a proof of Cauchy's ODEIVP existence theorem assuming only continuity of the ODE. Kennedy said that it was not completely rigorous, but was later fixed. I've very recently read that Peano's first woman PhD student—perhaps his very first PhD student—worked on this problem. (I believe the proof we use now is due to Émile Picard. I learned it as a graduate student in the second-semester ODE class at SFSU.)

ii. In `1890f` expanded and republished that paper, in French in a German journal, to reach a larger audience.

iii. `1890b` is Peano's shocking little paper on a space-filling curve, one of the first examples of fractals. It showed the world that the notion of curve that, even today, we learn in calculus classes does not accord entirely with our geometric intuition. Again, Peano wrote in French and published in Germany. Hubert Kennedy published an English translation of this paper; the course bibliography cites it.

b. In 1887 Peano married a lady from the art world. Picture: Kennedy, page 33. They never had children.

   c. In 1888 Peano published an book on geometric applications of calculus. Click here to see
      i. the title page and table of contents.
      ii. The work began with a chapter on logic. On page 5 of 93 Peano noted its similarity to algebra, and that it can *serve* many different kinds of research. (To a certain extent, this algebra of logic is now called *Boolean* algebra.)
      iii. On pages 78–79 of 93 Peano introduced to the world the definition of a vector space (*sistema lineare*). He was not quite explicit about what algebraic rules are postulated. This is preliminary work, familiar in spirit to our logical notation but seemingly awkward to us.

6. The last five minutes of the hour were devoted to the Department's class evaluation survey.