

1. The first part of this meeting was devoted to quiz 3.
2. *Assignment*
 - a. Continue reading Kennedy's Peano biography, through chapter 4.
 - b. Finish reading Struik, chapter 7.
3. *Mathematics students in Italian universities*
 - a. University mathematics students took classes during the first two years with students of other sciences. Then they studied "higher" mathematics for two years. Commonly they took one fewer course in the third and the fourth year, to compensate for the higher level and expected research. The higher courses were generally quite small. The professor often taught the third- and fourth-year courses together, varying the topics, so that "higher geometry," for example, would be a two-year course.
 - b. Students were examined at the end of each year, orally I think, but perhaps sometimes in writing, in several subjects, by a team of professors. I have the impression that the fourth-year examinations, at least in some universities, were formidable.
 - c. In Peano's time some universities required doctoral dissertations for graduation after the fourth year. Kennedy seems to have missed this detail for Peano, and the huge CD of Peano's published work does not identify any dissertation. On the other hand, Pieri, at a more prestigious university (the Scuola Normale Superiore in Pisa) a few years later, wrote *two* such dissertations! I have copies of both, but haven't studied them yet. I suspect that Peano may have written something comparable at the University of Turin, but it just hasn't turned up yet. A dissertation usually contained original research that was publishable, but not always.
 - d. When a mathematician earned the *laureate*—the term I use for the Italian degree to distinguish it from the rather stronger American doctoral degree—he usually took an examination for the license to teach in middle schools, too. Although the laureate qualified him for a university assistantship, there weren't many of those, and most laureates went immediately into middle-school teaching. These included the academic high schools (*licei* and *ginnasi*) as well as the technical, teacher-training, and military schools just below university level.
 - e. Most laureates stayed in those positions, and many continued to work on the side as researchers, sometimes *prolific* researchers. To earn the right to compete for a university chair, one had to study a few years more, and present published papers or other evidence to a government committee. Those successful were *habilitated*: they became *liberi docenti* (free instructors). They could be hired to fill chairs temporarily, and could compete for permanent chairs.

- f. During these years, the Italian universities were producing far more laureates than could be appropriately employed. Peano's fourteen-year quest for a chair in mathematics was excruciating. Peano was fortunate to avoid that harsh experience. The large population of underemployed highly educated Italians led to many political problems.
4. *Struik*, 7.6–7.8
- a. Euler was a major contributor to all fields of mathematics, pure and applied. To him are due our approaches
- i. to trigonometry,
 - ii. to exponentials and logarithms,
 - iii. to analytic geometry,
 - iv. to Newtonian mechanics, and
 - v. to much of our calculus curriculum.
- He published around a thousand works. His collected works, which began publication in 1916, are still under preparation in Switzerland.
- b. In Smith 2000, I cited his work as the source of
- i. the theorem that every three-dimensional motion—transformation that preserves all distances—has the form $x \rightarrow Ax + b$ where x is a variable coordinate triple, A is a fixed orthogonal matrix, and b a fixed coordinate triple, and
 - ii. the proof of theorem that for a polyhedron with V vertices, E edges, and F faces, $V - E + F = 2$.
- Both are absolutely fundamental to the study of geometry.
- c. In lecture 20 I showed some of Euler's handwritten personal notebooks. ([Click here](#) to review them.) As a high-school student I learned the German handwriting style common until World War II, so I could read them fairly easily. He used a funny combination of German and Latin, the latter when he needed a technical term that as yet had no common German equivalent. For publication, he wrote in simple Latin for "popular" consumption. His language in the excerpt in Struik is not hard to figure out. (Note that the modern editor is writing in Latin, too!)
- d. Euler's mathematics in Struik's example is somewhat slapdash, though. I think Euler is reasoning roughly as follows. (It helps to realize that his i stands for a natural number, not the imaginary unit. But I will use i for that unit here.)
- i. We know that for small real θ , $\cos\theta \approx 1$ and $\sin\theta \approx \theta$.
 - ii. And for real z and large integral n , $e^z \approx (1 + z/n)^n$,
 - iii. hence the same approximation should hold for $z = i\theta$ as well, and an analogous one for $z = -i\theta$.
 - iv. Moreover, $(1 + z/n)^n \approx 1 + n(z/n)$ and $(1 - z/n)^n \approx 1 - n(z/n)$.
 - v. You can put the previous approximations together to get $\frac{1}{2}(e^{i\theta} + e^{-i\theta}) \approx \cos\theta$ and $\frac{1}{2}i(e^{i\theta} - e^{-i\theta}) \approx \sin\theta$.
 - vi. And those two yield $e^{i\theta} = \cos\theta + i\sin\theta$.

How he argued that the approximation errors would cancel to yield equality, I don't know. I do know that we could keep more terms of the Taylor series beyond the first, and show that those cancel.

- e. Struik noted that some of Euler's flimsy arguments with possibly nonconvergent power series depended only on the algebraic properties of the coefficients, not on convergence, hence have been redone in modern times rigorously with the theory of formal power series.
- f. But Euler's treatment of infinitesimals, though more elaborate than Newton's and Leibniz's, was equally unfixable, and awaited the definition of limit, supplied by the Parisian encyclopedist d'Alembert.
- g. Struik's flouting of bizarre consequences supposedly derived from faulty reasoning with series should be ignored, unless perhaps as a reminder that from false premisses one can prove anything whatever.
- h. Struik mentioned the intellectual struggle between Cartesianism and Newtonianism. I'm not much into isms, but it seems that the former, an amalgam of Descartes' and his followers' doctrines, embraced both a philosophical point of view opposed by the Church, and some primitive pre-Newtonian physics, which was demonstrably wrong. Evidently the former had such appeal to intellectuals that they found it hard to dismiss the latter. So it took decades for Newtonian physics to supplant the Cartesian.
- i. This seems to be a good place to cite Thomas Kuhn's *Structure of scientific revolutions*, which is about how such turnovers come to pass. First published in 1962, that PhD thesis is one of the most influential pieces of graduate study ever. It presented scientific progress as often not gradual at all, but the result of sudden major upsets. It was required reading when my wife began graduate study at SFSU in social sciences around 1964, and probably still is. It would probably shed light on this question.