1. *Struik, section 6.6.*
   a. Struik tabulated when the most notable Greek works became available in Latin. With the increasing facility in algebra, mathematicians produced a flood of new works anticipating calculus.
   b. Struik mentioned the start of probability theory with Pascal. I'm sadly ignorant of that area of mathematics, and won't try to cover it.
   c. *Mathematical induction*.
      i. Struik noted that Pascal used our "modern" formulation of mathematical induction: if $P(n)$ is a statement about a natural number n then we can prove the statement "for all n, $P(n)$" by
         (1) first proving the statement $P(1)$, then
         (2) proving the statement "for all n, $P(n)$ implies $P(n+1)$".
      ii. Struik referred to Freudenthal 1953. That paper actually refutes work of several historians whose claims that other mathematicians used the modern form quite early were not justified. Some of the discredited studies are in journals that Internet searches will find readily: Beware! This subject comes up later, in the 19th-century work of Grassmann and Peano.
      iii. Struik also referred to Rashed 1983, an example of the practice of history partially for the enhancement of the image of a cultural group. Rashed rejected Freudenthal's claim that Pascal was first, and made the case for some much earlier mathematicians who wrote in Arabic. I have no way to judge the matter, only to note that Freudenthal is certainly Eurocentric and Rashed, an exponent of the Muslim culture. Rashed is probably the leading historian of ancient Muslim mathematics.
      iv. Freudenthal was a noted geometer, with interests in many other areas, such as linguistics, history, and mathematics education, and a wonderful writer. My first paper Smith 1973 on foundations of geometry was published, at his suggestion, in the first volume of a journal that he was editing.
   d. Pascal's theorem about the hexagon says that the three pairs of opposite edges of a hexagon inscribed in a conic section, when produced, meet in three collinear points. One proves it first for a circle, then projects the figure so that the circle is mapped onto the conic.
   e. The case where the conic is degenerate, consisting of two lines, was known 1200 years earlier. It's called *Pappus' theorem*, and is used as an axiom in projective geometry, because of its close association with the commutative law of multiplication. (That was discovered only in the 1890s, though.)
   f. The dual of Pascal's theorem is rather prettier: the diagonals of a hexagon circumscribed about a conic concur. I think it was discovered about 200 years later. It's known as *Brianchon's theorem*.

g. Struik mentioned Desargues, another contemporary of Descartes and Pascal. He was a military engineer. I'll draw *Desargues' theorem*, which is another axiom for projective geometry, in fact more fundamental than Pappus'. The theorem was originally formulated to handle problems in perspective drawing. See the discussion in Smith 2000, chapter 1.

h. Proofs of Pappus', Desargues', and Pascal's theorems in Euclidean geometry are rather difficult.

2. *Struik, sections 6.7–6.8*

a. This calendar from Struik is useful:

	Newton	*Leibniz*	*Leibnizians*
1642	Born.		
1646		Born.	
1665–1666	Calculus, light theory, physics developed.		
1669	Prof. at Cambridge.		
1673–1676		Calculus developed.	
1684		Calculus published.	
1687	Physics published.		
1690			Bernouillis get started.
1696	To the mint.		l'Hôpital calculus text.
1704	Cubics published.		
1704	Optics published.		
1704–1736	Calc published.		
1714		George I to England.	
1716		Died.	
1727	Died.		

b. This is the time when England ran out of royalty, and recruited the king of the German state of Hanover to become king George I of England. The countries remained partially united until the 1800s.

i. Germany then consisted of *very many* small countries and a couple of bigger ones.

ii. At that time Leibniz was working for the government in Hannover, and Newton for the government in London.

iii. That George was the grandfather of George III, who reigned during the American revolution.

c. "Developing calculus" means

i. understanding the fundamental theorem of calculus,

ii. formulating techniques for its application.

d. Newton and Leibniz worked independently.

e. Click here for some pictures of Trinity College, Cambridge, where Newton worked.

f. Leibniz came upon calculus while in Paris, as a (very young) diplomat for the Elector of Mainz. He was brilliant, curious about everything, and evidently had a lot of time on his hands to pursue his interests.

i. Mainz, a city on the Rhine in central Germany, was independent at that time. With the surrounding area, it was governed by an Archbishop of the Catholic Church, who was one of the very few who were privileged to elect the Holy Roman Emperor, generally the Habsburg emperor of Austria.

g. Newton's "physics" here means his formulation of the concepts of force and gravitation, of general principles governing them, and his derivation from them of Kepler's experimentally derived laws of planetary motion. Newton's physics explained the observations.

h. Newton's example on Struik, 107, is one of the steps in what is now known as implicit differentiation. He considered small increments dt of the time variable (he wrote o for dt), did simple algebra that required, $dt \neq 0$, then regarded $dt^2 = 0$, and ended up with the correct formula for the derivative in that example. That there is a problem with consistency here is evident.

i. The foundations of Leibniz' calculus were as shaky as Newton's, in the same way.

j. Really good mathematicians could usually apply Newton's and Leibniz' calculus correctly to the applications and theoretical problems they encountered for the next 150 years. Sometimes they went astray because of the underlying vagueness. Lesser minds probably strayed more often. But by 1850 or so, serious reconsiderations were required to make further progress.

k. Newton's approach emphasized motion, Leibniz' emphasized static geometry.