1. *Assignment*: read Struik, chapter 7.
   a. I'm going to pick material from that chapter and the following one specifically to lead to the calculus courses that Peano would have taken at the *liceo* and university in Turin around 1876, and which he started to teach immediately after his degree.
   b. This causes me—with some relief—to avoid considering the development of probability theory. I know little about that area, and Peano didn't teach or research in probability theory.
2. *Struik, sections 6.3–6.4*
   a. Struik stressed that Descartes, in his 1637 *Discourse on method*, was searching for a rationalistic way of thinking and problem-solving, and for a mechanistic way of understanding the universe.
   b. It was written in French to gain a larger audience than Latin would attract.
   c. Descartes lived in troubled political times, fled persecution, was involved with military operations, etc.
   d. Descartes' [1637] 1954 appendix, *Geometry*, popularized a more modern notation, the elementary uses of coordinates, and their relationships to some hard algebra problems.
   e. Descartes also discarded the practice of adding only like quantities, which had discouraged algebraic expressions such as $x^3 + x^2 + x + 1$, which would earlier have been interpreted as the sum of a solid figure, a plane figure, a line segment, and a number.
   f. Struik noted that the first European academies of science, in Rome, London, and Paris, were founded in the era 1580–1670
   g. The Italian academy is *L'Accademia Nazionale dei Lincei*. The word *linceo* means *lynx*, or *wildcat*!
   h. Note that the time of Descartes, Cavalieri, Fermat, et al., coincided with the start of higher education in this country: Harvard was founded in 1636 as an institution for training clergy for the Massachusetts Bay colony. (Too many students that they sent to England for training failed to return!)
   i. Struik noted that other mathematicians of that time and earlier had applied algebraic and coordinate methods to conics. They knew that conics had quadratic equations.
   j. Moreover, they began to anticipate *differential calculus*.
      i. For example, you can figure out that the line through points $<x_0, y_0>$ and $<x_1, y_1>$ on the graph of the equation $y = ax^2 + bx + c$ has equation $y = y_0 + (x - x_0)[a(x_1 + x_0) + b]$. When $x_0 \neq x_1$ this is a secant line. As $x_1$ approaches $x_0$ it approaches the line that touches the curve at that one point $<x_0, y_0>$ only: the tangent line. In that case the line has equation $y = y_0 + (x - x_0)(2ax_0 + b)$. Its slope is $2ax_0 + b$, the *derivative* of $ax^2 + bx + c$ at $x = x_0$.

ii. With somewhat more complicated algebra, this method will yield the derivatives of the higher integral power functions.

k. I learned some of these precursory approaches to calculus from Otto Toeplitz's wonderful [1949] 1963 text *Calculus: a genetic approach*.

3. *Struik, section 6.5.*

a. In the time under discussion oceanic commercial and naval rivalry was paramount. Success depended on navigation, and navigation depended on the determination of longitude. It's easy: just figure out the number of hours between your sunrise and the most recent one at Greenwich, and multiply by $360°/24$ to get your west longitude. But that requires your keeping with you a clock on Greenwich time. So success depended on good clock design.

b. Mathematicians investigated curves that might contribute to clock design.

c. The *cycloid* is drawn by a point $P$ on the rim of a wheel with center $C$ rolling at a constant angular speed along a horizontal axis. (*Contra* Struik, it's not called a *roulette*; that's something else.) Since it meets the axis at infinitely many points, the curve cannot have a polynomial equation. Suppose $P$ starts at the origin of the axis. Its motion relative to the axis is the composition of its rotation about $C$ with the translation of $C$ along the axis. These have parametric equations

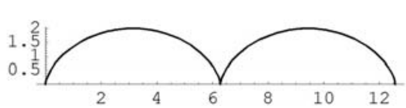
$$\begin{cases} x = \cos\left(-\theta - \tfrac{1}{2}\pi\right) \\ y = \sin\left(-\theta - \tfrac{1}{2}\pi\right) \end{cases} \qquad \begin{cases} x = \theta \\ y = 1 \end{cases}.$$

(The direction of rotation is negative, and $x, y = 0, -1$ relative to the center when $\theta = 0$. To avoid slippage, the center must move $2\pi$ units horizontally during each rotation of the wheel.) Use the identities $\cos(-\theta - \tfrac{1}{2}\pi) = -\sin\theta$ and $\sin(-\theta - \tfrac{1}{2}\pi) = -\cos\theta$ to get the parametric equations

$$\begin{cases} x = \theta - \sin\theta \\ y = 1 - \cos\theta \end{cases}$$

for the motion of $P$ relative to the origin. This *Mathematica* code produced the displayed graph:

```
ParametricPlot[{θ-Sin[θ],1-Cos[θ]},{θ,0,4π},
    PlotStyle→AbsoluteThickness[1],
    AspectRatio→Automatic];
Export["d:\\Math300\Week09\Cycloid.bmp",%,
    ImageResolution→300];
```



d. Think of a cycloid $\mathscr{E}$ upside down with a string half the length of one arch attached at one of the cusps. Stretched taut and swung about, it will trace another such arch $\mathscr{I}$. (That's not at all easy to show!)

e. Weighted at its free end the string forms a pendulum. With no friction and no mass for the string itself, the period of the oscillation is independent of the amplitude: that is the *isochrone* property of the cycloid.

f. Applied to any sort of curve $\mathscr{E}$, that process produces a curve $\mathscr{I}$ called the *involute* of $\mathscr{E}$. So, a cycloid is is own involute. The involute of a circle is a kind of spiral. Involutes are used in the design of the teeth of gears.

g. In turn, $\mathscr{E}$ is called the *evolute* of $\mathscr{I}$. In general, the evolute of a curve is the locus of its centers of curvature: that result is often the most involved bit of curve theory covered in calculus classes. Clearly, a cycloid is its own evolute, and the evolute of a circle is a single point.

h. The inverted cycloid also has the *tautochrone* property: a friction-free bead sliding down the curve will reach the lowest point in the same time, no matter where it starts.

i. Finally, it has the *brachistochrone* property: of all curves, the cycloid provides the path for which that sliding time is least.

j. Struik mentioned the *tractrix*. If you take that weighted string, place the weight on a table top, stretch the string so that its other end is on the edge, and pull it along the edge, the weight will follow a curve called the *tractrix*. Surprise: it is the involute of a catenary.

k. Struik also mentioned a ‘logarithmic’ curve. That's vague. He probably meant the logarithmic, or equiangular, spiral.

l. These studies were a great challenge to the mathematicians of the time. Now they're just at the limits of undergraduate calculus classes.

m. You could find a wealth of term paper topics in the areas mentioned in this item!