

1. *First term papers*
 - a. The first set of term papers came out quite good from my point of view: *Congratulations!* Here is the grade distribution:
 - i. 80 ... 100% xxxxx xxx
 - ii. 65 ... 80% xxxxx xxxxx
 - iii. 50 ... 65% xxxxx xx
 - iv. 40 ... 50% xxx
 - v. 0 ... 40% x
 - b. Here are the titles of the top eight papers:
 - i. The Development of Ehrhart Theory
 - ii. Two Historic Figures in Math and Science: The Genius of Pascal and Leibniz
 - iii. Alicia Boole Stott: A Diamond in the Rough Mathematician
 - iv. Demography: A Look Back at Ourselves
 - v. The Influence and Networking of Alfred James Lotka
 - vi. History of Hindu Mathematics
 - vii. Mathematics from Islamic Societies: Al-Khwarizmi and Omar Khayyam
 - viii. Fractals: A History of Dimensional Curiosity
 - c. Three criticisms were most common in my reviews of the papers.
 - i. *Lack of organization.* Most papers lacked visual organization into sections with headings. Moreover, the condition of many seemed to indicate that the authors did not employ any *outlining*. I outline *everything!* That way I always have a framework—title, introduction, body, conclusion, references—to start with. I fill in subtopics and sub-subtopics gradually, leaving placeholders for material that I still need to look up or formulate. I leave collapsing the outline into connected prose until the very last minute. It's often only a minor effort.
 - ii. *Word processing.* Learn the detailed controls of your word processor. It will handle most prose-production techniques with great ease, if you merely find the controls! The way to learn that is to investigate *all* those pull-down menus at the top of the screen.
 - iii. *Use appropriate sources.* I still complained about use of sources intended for audiences who have not studied any university-level mathematics—even literature for grade-school children. These are often by writers who have little background beyond yours, and consequentially prone to misinformation.
2. *Real-world writing*
 - a. My current major project besides this course is to edit an article, “Definitions and nondefinability in Euclidean geometry” for publication in the *American Mathematical Monthly*. Smaller versions of this work have been given at SFSU, Whiskeytown, and Turin, and the last will be published in Turin soon.

I showed the dressed-up version of the paper that I submitted to the *Monthly* in January. Last month I received an email from the editor, with two anonymous referees' reports. The editor said that the paper would be acceptable provided I address some major complaints from the referees. I showed one of the reports, which is often not at all polite! The referees found several serious mistakes, and their complaints about balance and questioning of stated facts indicated that I had not done a good enough job of exposition. I showed my own document that contains my responses to their comments: corrections, rephrasings, and additions to the submitted paper. Then I showed my partially complete version of the submitted paper with corrections interposed in blue, ready for execution.

- b. One student asked, what gives that nameless person the right to say such things? I replied that I belong to the Mathematical Association of America, and thus participate in the election of its officers. They in turn appoint the editor, and he solicits volunteer help from professors as knowledgeable as I, or more so, who serve as referees.
 - c. Prof. Beck recently underwent the same process. He told me that should the paper be accepted, it will still undergo severe copyediting by the *Monthly* staff before it gets published. It is *very* hard to publish in that journal!
3. *Geography*. One reason for the stress on geographical distinctions in the quizzes is the *international* nature of *current* mathematics. For example, the current roster of the twenty-one tenured or tenure-track professors in our Department includes individuals stemming from *sixteen* countries! Five others, on administrative duty and/or retired, are from the U. S.
 4. *Struik, sections 6.1–6.2*
 - a. Kepler could derive the formula for the area of a circle quickly by dividing it into sectors and approximating them by triangles. He applied such methods to practical calculations of volumes of barrels, etc.
 - b. Also a virtuosic calculator, Kepler discovered some famous laws of physics by analyzing Tycho Brahe's detailed observations of planetary motions.
 - i. Planets' orbits are elliptical with the sun at a focus.
 - ii. A line from the sun to a planet will sweep out equal areas in equal times.
 - iii. The period of a planet's revolution about the sun is proportional to the 1.5 power of its distance from the sun.
 - c. Kepler also studied the efficiency of various ways of stacking balls of a given radius in a large box: the limit of the ratio of the total volume of the balls to the volume of the box, as the dimensions of the box increase without bound. He knew that the method used to stack oranges or cannonballs, each one lying in the cup formed by three balls below, is more efficient than the one which puts each ball tangent to just one ball below. Kepler conjectured that the cannonball method was the most efficient. This question was under intense study for centuries. About 25 years ago one affirmative solution was circulated, but its author couldn't justify his arguments sufficiently, and his work received no further attention. About 10 years ago, Thomas Hales submitted

an affirmative solution that has met general acceptance. However, his work required computer processing of very many special cases, and no journal has agreed that its referees were able to check the computer work. The mathematics in both of those solutions is simply detailed elementary geometry and trigonometry; but there are hundreds of pages of it. Mathematicians await a more easily understood solution.

- d. Galileo set the scene for experimental science with his studies of motion and mechanics.
- e. In his 1635 book Galileo's student and secretary Cavalieri anticipated the development of calculus in several ways. In particular he formulated the principle that if two solids are enclosed by distinct parallel planes α and β and planes parallel to and between those cut the solids in plane figures with equal area, then the solids have equal volume. That amounts to taking as a postulate a principle stronger than what Euclid proved for pyramids by the method of exhaustion. It is often used as a postulate in current school geometry texts.
- f. As shown earlier in class, integral calculus yields an even simpler argument. Mathematicians continued to wonder, however, whether calculus, the method of exhaustion, and Cavalieri's principle could all be avoided. The negative answer was provided by Max Dehn (the reviewer of our text) in 1903.