1. *Assignment*: Struik, chapter 6.
2. *Struik, sections 5.1–5.3.*
    a. The eastern Roman, or Byzantine, empire dates from about 400 C.E.
    b. Struik mentioned the seven liberal arts of the medieval European university tradition. He referred to Boethius but didn't say they originated with him (I had to read that twice). I don't know their history. But their tradition persists in our liberal arts education system.
    c. *Carolingians* = the dynasty of Holy Roman emperors founded by Charlemagne (742–814 C.E.).
    d. Struik noted that in Western Europe the landed aristocracy rose as commerce fell and large-scale economies vanished. The agricultural base of Western economies made bureaucracies unnecessary, and those had supported the early development of mathematics. Eventually the towns emerged as self-governing. The first cities to establish, or re-establish, commercial ties with Eastern civilizations (including Arab) were Italian, then French. In Spain and Sicily were contact points,
    e. particularly Toledo, after its reconquest by Christians in 1085. There (and elsewhere) scholars, often Jewish, translated into Latin the Arabic texts, which often stemmed from earlier Greek work. Struik claimed that by this time the West was advanced enough intellectually to absorb this material.
    f. Several years ago, I attended a conference in Granada, Spain, and was impressed by today's efforts to sort out the history of this process. Scholars of Arabic and Latin sources, many of North African descent, are writing and speaking mostly French.
    g. There was also some rather fascinating work on the development of mathematics at that time in Catalan, the first Romance language to support a mathematics literature. Catalan is used in Barcelona, and modern university mathematics texts are being published in that language. These studies are to a certain extent aimed at enhancing world awareness of Catalan culture.
    h. Struik noted that in Constantinople, in the Eastern Roman empire, it was still possible in the times under consideration to study the original Greek works.
3. *Cross-cultural influences in ornamentation*. We've already seen Roman ornamentation clearly derived from Greek geometry. I don't know about Roman/Greek influence on early Christian ornamentation. But with the Muslim conquests from about 700 C.E. the cross-cultural influences were flamboyant! Click here for some pictures of that: the great mosque in Córdoba, Spain, which is now its cathedral, the Norman royal chapel in Palermo, Sicily, and the cathedral in nearby Monreale, and finally one of the culminations of Muslim ornament, the Alhambra palace in Grenada, Spain.

4. *Struik, sections 5.4–5.5.*
   a. Struik noted the contact from about 1000 C.E. between Northern Italy and the Orient: Marco Polo, for example. A banking system was developed.
   b. Leonardo of Pisa = Fibonacci (1170–1250 C.E.) was a merchant and mathematician.
      i. Fi = son of, as in the Irish Fitzgerald; Bonacci is a family name.
      ii. Each of the Fibonacci numbers $1, 2, 3, 5, 8, 13, \ldots$ after the second is the sum of the preceding two. Fibonacci introduced them in studying the growth of a population. They play important roles in many areas of mathematics, both entertaining and practical.
      iii. We've already seen that he demonstrated that every positive rational number can be expressed in Egyptian form.
      iv. Struik said that Fibonacci proved that solutions of some cubic equations with integer coefficients cannot be expressed by formulas involving just the operations $+, -, \times, /,$ and $\sqrt{\;}$. That sounded terribly interesting, so I started looking it up. I consulted Moise 1990, section 19.10, and found the theorem that such a cubic must have a rational root if it has one expressible with $\sqrt{\;}$. Burton (2002, §6.2), showed how Fibonacci proved that one cubic of this form has no rational root and verified that it has no root with formulas of certain specific types involving radicals. Then Burton made the same statement as Struik, but did not indicate how Fibonacci might have extended his argument to cover *all* such formulas. Moise's argument depends on algebraic techniques developed during the 1800s, so I rather doubt that Fibonacci had a complete proof. I don't have time to research this further. It would make a good topic for an expository paper in our master's program.
      v. Evidently Fibonacci also pioneered the European use of the Hindu-Arabic positional numerals. They did not become universal in Europe, however, until Renaissance times.
      vi. The Fibonacci society, headquartered at San Diego State University, publishes a *Quarterly* journal devoted to number theory in the Fibonacci tradition. Prof. Robbins has published a number of papers there.
   c. Struik noted that in these times mathematics was largely taught, developed, and practiced outside universities, in a context that might be analogous to a trade school or business consulting.
   d. Speculative theoretical mathematics was rare and done mainly in the religious schools of the scholastic tradition. Struik mentioned some writing about the question of "actually" versus "potentially" infinite sets, which stemmed from the Greeks and would play a large role in theoretical mathematics from about 1700 on.
   e. Oresme, around 1480, was using coordinates in geometry, and knew that the harmonic series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$ diverges.

5. *Struik, sections 5.6–5.7*
    a. Struik noted that during 1400–1600 Nuremberg in Germany, Prague (now in the Czech Republic), and Vienna in Austria became major centers of commerce and learning. After 1453, when Constantinople fell to the Turks, many Greek scholars moved from there to central Europe.
    b. Struik noted the 1500s work of Johannes Müller, a German also known as Regiomontanus, who developed highly accurate trigonometric tables, etc., for commercial use.
    c. The first major step beyond the Hellenistic and Arab mathematics, according to Struik, was the 1500s work by the Italians, at Bologna in particular, on algebra, and in several locations on perspective drawing.
6. I wrote long ago some notes on the solutions of the general cubic and quartic equations, described by Struik. They are aimed at students very adept at high-school algebra and trigonometry. Click here to read them. Their last two paragraphs cover some of the history of the Italians' work, which is wild and wooly. I didn't attempt to adhere to the original mathematical reasoning: that is difficult to do in a mathematics class because those Italians still lacked algebraic notation.
    a. In class I'll solve the equation $x^3 + px - q = 0$.
    b. The Italians' achievements in solving cubics and quartics stemmed in part from challenges posed between competing "scientific consultants." Of course, these continued: solution of the general quintic equation was one of the major challenges in mathematics during the next centuries. Only in the 1800s was it shown that such solutions are not possible using just the familiar formulas of algebra.
7. Struik emphasized the Italians' and Dürer's work in perspective drawing. It's closely tied to Renaissance developments in art.
    a. Artists' clients began to prefer realistic painting: their eyes should see something that would remind them of reality, rather than insulate them from it.
    b. Click here for two fine examples of perspective painting by the Northern Italian mathematician and artist Piero della Francesca, completed around 1455 and 1473. It's fun to contrast this work with attempts at such scenes by earlier painters, and even more to see how the new techniques were implemented in Northern Europe only gradually over the next centuries. Artists competed in depicting very complex scenes in perspective.