

1. *Quiz 2* will be structured like quiz 1, and will cover material referred to in lectures up to the end of today's. I won't ask specific questions about material covered by quiz 1, but may expect you to relate some of that to material covered later.
2. *Assignment*. Read Struik, chapter 5.
3. *Struik, chapter 3, concluded*.
  - a. *Sines*. Our standard  $\sin\theta$  is half the chord of the unit circle subtended by twice the angle  $\theta$ . The only major difference between us and the ancients in this regard is that they didn't standardize to the unit circle, and thus tabulated sines for various radii. They used large radii to avoid calculating with fractions. Also, Ptolemy used the whole chord, not half. Struik regarded the tabulation of sines as yet more evidence of the continuation of Oriental, computational, strains in Alexandrian mathematics.
  - b. The Greek influence, Hellenism, spread to India, where the earlier work was also extended. Cite Brahmagupta's theorem, from around 600 C.E. as an example: the area of a cyclic quadrilateral with edge lengths  $a, b, c, d$  is the square root of  $(s - a)(s - b)(s - c)(s - d)$ , where  $s$  is the semiperimeter. A *cyclic* quadrilateral is one that can be inscribed in a circle. You can obtain Heron's formula for the area of a triangle  $T$  by noting that  $T$  can always be inscribed in a circle, and regarding it as a quadrilateral with one edge of length zero.
  - c. Struik noted that the Roman conquest of the formerly Greek areas, starting around 200 B.C.E. and continuing for 200 years or more, led to the rise of "extensive" agricultural economies in the West based on slavery, and this generally removed the motivation for scientific advance. The "intensive" agriculture of the East continued, and the Eastern civilizations continued some slavery. Greek mathematics waned more slowly in the East.
  - d. [Click here](#) for pictures that illustrate virtuosic Roman application of Greek geometric techniques in the design of the mosaic floors of a 300 C.E. villa in Sicily. This villa was the house of a very rich Roman who ran one of those "extensive" farms.
  - e. Struik pointed out that the long period of relative peace in both the Roman and Chinese civilizations at this time promoted a transfer of scientific ideas in both directions.
  - f. Struik described the inefficient nonpositional numeration system of the Greeks. He didn't say it inhibited mathematical development, but it certainly didn't foster development of computational techniques.
4. *Struik, chapter 4*
  - a. *Positive numbers only*. How unnecessarily complicated the early algebra was, simply because the ancients didn't admit negative numbers! I find this astounding: negatives didn't come into common use until the late Renaissance times.

- b. Decimal positional notation came to us from the Arabs around 600 C.E., to them from the Hindus, and perhaps to the Hindus from the Chinese, though apparently we don't really know about the very earliest history. Our Latin digits are somewhat like modern Arabic ones. Here are the latter from *Word-Perfect's* built-in Arabic typeface: 0123456789 = ٠١٢٣٤٥٦٧٨٩.
- c. Al-Khwarizmi's name and the title of his book gave us the terms *algorithm* and *algebra*. It wasn't really algebraic yet, but like the Oriental works in general, stressed practical computation over theory. *Khwarizmi* is a place-name: the old version of the name of *Khiva*, a city in Uzbekistan.
- d. We'll see how algebraic notation and manipulations emerged. But, as Struik noted, algebra was never developed axiomatically (that is, from rules such as the associative and distributive laws) until the late 1800s. As we'll see, Giuseppe Peano was heavily involved in that.
- e. Struik noted progress in the formation of trigonometric tables and spherical trigonometry. I used to present in trigonometry classes the methods used by the Alexandrian Greeks to get very close approximations to the trigonometric functions. My source was Van der Waerden's marvelous 1963 *Science Awakening*, which also detailed the development of spherical trigonometry by the Alexandrians.
- f. Struik mentioned the Arabs' translations of Greek mathematical manuscripts. These were the principal means of our learning this mathematics.
- g. Struik mentioned that Omar Khayyam was concerned with solving cubics. We saw that with the earlier Greeks, and will see it again with the Renaissance Italians.
- h. Omar's sophistication in Euclidean geometry is news to me. I'll have to find out about that!
- i. *Calendar reforms*. You should be familiar with the difference between our old Julian and current Gregorian calendars (named after popes). The Gregorian reform was proposed in late Renaissance times, to drop about two weeks because the calendar had become inconsistent with the seasons, and to institute the present leap-year system. Its adoption took centuries, with the last conversions triggered by the Russian revolution. Those two weeks cause no end of trouble to historians: they were dropped in the 1700s in this country, and dates in that era are often inconsistent.
- j. I don't know enough about the other calendars in common use to write about them.
- k. *Toledan* = having to do with Toledo, the city in Spain.
- l. Struik very briefly mentioned early mathematics in China. There was little research on that subject for him to digest. It is only now being studied in depth, and that is extremely difficult due to the scarcity of preserved materials.