1. *Fong Symposium*. Prof. DeWitt Summers of Florida State will give the Fong lectures 18–19 March, on mathematical biology, and the topology of DNA. The first is aimed at the scientifically literate public, the second, and mathematics undergraduates. Click here for a flyer.
2. *Platonic solids, concluded*
   a. Euclid sketched a proof that there are *at most* five polyhedra whose faces are all regular polygons, with the same number of faces at each vertex, and the same angles between adjacent faces. Click here for pictures.
   b. It's easy to show that the cube and regular tetrahedra and octahedra actually exist, but not so easy for regular dodecahedra and icosahedra. It's insufficient to just make models, because you can't tell whether the angles match and faces intersect exactly. I believe that Euclid made some attempt at that.
   c. To execute those drawings, I used this method, which constitutes a proof:
      i. The points with coordinates $(\pm 1, 0, 0), (0, \pm 1, 0), (0, 0, \pm 1)$ are the vertices of a regular octahedron. It has twelve edges.
      ii. On each edge find two points, each of which divides the edge into two segments whose length ratio is golden.
      iii. From each of those twelve pairs you can select a single point, so that the distances between each selected point and its nearest selected neighbors appear equal. Then verify, using coordinate geometry, that they all are.
      iv. The selected points are the vertices of a regular icosahedron. Use coordinate geometry to check that the angles are equal.
      v. The centers of the triangular faces of the icosahedron are the vertices of a regular dodecahedron.
   d. Ms Howard attended the MAA meeting last week at MSRI and was presented a T-shirt. On her shirt is depicted an alternate existence proof: arrange three congruent golden rectangles so that their axes are mutually perpendicular and they share the same center; their twelve vertices are the vertices of a regular icosahedron.
3. *Greeks*
   a. We've been studying work of the golden age of Greek culture. Very little evidence has survived of earlier Greek mathematics. But the work of the golden age was so sophisticated that it must have taken shape over centuries. During that time the Greek culture was spreading around the Eastern Mediterranean. Pythagoras had moved to Croton, in Southern Italy and started a philosophical school there by about 500 B.C.E.
   b. Athens became supreme by 430 B.C.E., having repelled Persian attacks, then leadership passed to other city-states. During 400–300 B.C.E., Plato and Aristotle flourished in Athens, Alexander extended Greek domains to Central Asia, and Alexandria was founded in Egypt. It became a great center of

mathematics. Archimedes flourished around 250 B.C.E. in Syracuse on the island of Sicily. Click here for a map of these places.

c. Click here for some pictures of the remains of the Greek settlement at Agrigento on the South coast of Sicily, which dates from this era. Also visible are remains of later Christian burial sites, and the city that developed and survived on the hill above.

d. Click here for some sights in Syracuse. The cathedral, recently cleaned, incorporates the temple of Athena, built before 400 B.C.E. Its baroque exterior dates from the 1700s.

4. *More on Struik, chapter 3*

   a. Struik noted that practically all classical texts used a rigorous but rather sterile approach, not a looser but more fertile one that was beginning to appear. He felt that the sterility was connected with mathematics' becoming (again?) the hobby of the leisure class.

   b. Struik noted that from Euclid on, there were professional mathematicians.

   c. Euclid, around 300 B.C.E., presented the theory of π. He proved that all circles have the same ratio of circumference to diameter. That required first relating the circumference to the perimeters of polygons. The letter π for this ratio was introduced 2000 years later.

   d. Archimedes, a little before 200 B.C.E., could compute π as accurately as anyone might demand, time permitting. He also had the formulas for the area and volume of a sphere. (Did Euclid?) Archimedes also is noted for practical work in mechanics and hydrostatics. Struik regarded that as an example of the continuation of the Oriental tradition in spite of the Greek dominance.

   e. Struik alluded to a number of Greeks, mainly at Alexandria, who elucidated and extended Euclid's work.

      i. Appolonius, contemporary with Archimedes, developed much of our theory of conic sections. There is a controversy about whether his work was a precursor of analytic geometry.

      ii. *Note: here we pass from B.C.E. to C.E.* After this lecture, I'll omit the designation C.E.

      iii. Around 100 C.E., (another) Ptolemy, Menelaus, and Heron contributed to geometry, spherical trigonometry, astronomy, geography, and engineering, and also started using longitude and latitude. Cite Menelaus' theorem, Heron's formula. "Heron" is sometimes spelled "Hero."

      iv. Diophantus, around 250 C.E., studied rational roots of multivariate polynomials with rational coefficients. That there is no algorithm for determining whether such roots exist was settled about 30 years ago.

      v. Pappus, around 300 C.E., was among the last Alexandrian mathematicians. He worked on geometry and engineering, and wrote a major commentary on Euclid. Cite Pappus's theorem. Its generalization to conics was discovered about 1300 years later. Pappus's theorem is often taken now as a postulate of projective geometry.