

1. *Example of inaccuracy*

- a. Take great care when using single sources for historical information, particularly Internet sources. They're less thoroughly refereed than published sources. Moreover, once misinformation is posted on an easily accessible source, it propagates. Here is an example that should embarrass several authors.
- b. Browsing for some examples for my course about writing mathematics, I came upon the paper Johnson 2002, "The curious history of Faà di Bruno's formula." The formula gives the m th derivative of the composition of two functions in terms of the derivatives of the components. I knew a little about Faà di Bruno from researching Mario Pieri, so I searched Johnson 2002 for biographical information. On page 232, after complaining of the inadequacy of the references in Faà di Bruno's papers, Johnson wrote,

Faà di Bruno has been the subject of several Italian biographies focusing on the religious aspects of his life. He was ordained a Roman Catholic priest on October 22, 1876, was a pioneer in charitable works, and was declared a Saint on September 25, 1988. In his last two papers for Sylvester's *American Journal of Mathematics* he was "the Rev. Faà di Bruno" and "l'Abbé Faà di Bruno". One might only wish that this admirable man had either read more or been a little more saintly in his citations.

He who lives in a glass house and throws stones should beware! I was suspicious, so checked other websites that might have better information. The St. Andrews website [O'Connor and Robertson \[no date\]](#) says the same thing about sainthood. I suspect that's where Johnson got his information. An [American Catholic website](#) says, "He died in Turin on March 27, 1888, and was beatified 100 years later." An [Italian mathematical biography website](#) says,

Da allora in poi, pur continuando ad insegnare, fu prevalentemente un sacerdote, fondatore, fra l'altro, di non poche 'opere' di assistenza. Giovanni Paolo II, nel 1988, a 100 anni dalla morte, lo ha beatificato.

Faà di Bruno has only been *beatified*, the first step to sainthood. That makes a big difference in Catholicism. Hubert Kennedy wrote somewhere that he had made the same error earlier, and others had copied his mistake. The American Mathematical Society's online *MathSciNet* reviews of Kennedy's book and Johnson's paper failed to report the error.

2. *Wikipedia*

- a. Once notoriously unreliable, *Wikipedia* is getting better. It's a good way to start on a search for historical or mathematical information.
- b. But it's *ephemeral*: it can change suddenly without warning.
- c. I plan to demonstrate that later by showing in class how to change the entry for Mario Pieri, on whom my coauthor and I are the authorities. I've never done that before, so it may be humorous.

- d. Because it's ephemeral, *Wikipedia* is useless for documenting your term papers: there's no guarantee that I could backtrack to the same information.
 - e. But it is ok to say something like, "*Google* and *Wikipedia* led me to the following resources. According to [Resource 1],..." Or this: "*Wikipedia* reported that John Doe died in 1898 but Jane Jones was born in 1896. Thus they probably had no professional contact." But be sure you've got the right John and Jane!
3. *Struik*, chapter 3: *Euclid*. I'm going to skip over some topics that Struik treated, even though they come before Euclid chronologically. This includes Pythagoras' theorem, on which a student will report. I'll return to that thread after we consider some more elementary material in Euclid.
- a. We know almost nothing about Euclid except that he wrote some books. His *Elements* is an organization of much that was known about geometry and number theory. We know it only through editions long after his time, and through others' comments about it.
 - b. Heath's classic translation Euclid [1908] 1956 is online: [click here](#). Other versions, perhaps faster to use but not hyperlinked, may be available too.
 - c. Heath was not a professional academic, but a government bureaucrat. Nevertheless, his edition is authoritative. His language is florid Victorian academic English, decidedly old-fashioned even when he wrote it. Many critiques have been published about it.
 - d. A more modern translation by David Joyce is also online: [click here](#). I don't think this has been published in hardcopy. Joyce is a professor at Clark University. I haven't read any critiques of his work.
 - e. Some of Euclid's "definitions":
 - i. (i) point
 - ii. (ii, iii) line and straight line
 - iii. (viii-x) angle, rectilineal angle, and right angle
 - iv. (xv,xvii) circle, center, and diameter.
 - f. *Line* was used in Heath's time as it is now, but also with the meanings that we would render now as *line segment* and *curve*. More modern writers often unfortunately follow Heath's usage in this context. Euclid may have used quite different terminology.
 - g. Euclid evidently left undefined the notions of equality of line segments and of angles.
 - h. Some of his "definitions" are not informative, and there may be some circularity. He did *not* follow Aristotle's guidelines.
4. *Postulates* as well as definitions underlay Euclid's geometry:
- a. There is a line (segment) between two points.
 - b. A line segment can be extended continuously.
 - c. There is a circle with any center and radius.
 - d. Right angles are equal.
 - e. Parallel postulate (deferred until a later discussion).
5. *Theorems* that Euclid derived from the postulates start with these.

- a. Proposition I.1: to construct an equilateral triangle on a given base.
- i. Ms. Ngo presented Euclid's proof: use postulate (c) to construct two circles with the given base as radius and its ends as centers; their intersection is the third vertex of the required triangle.
 - ii. Euclid's postulates do not justify the existence of intersection of the circles in the proof of proposition I.1.
 - iii. Modern, "cleaned-up" versions of Euclid often take the existence of that intersection as another postulate.
- b. Proposition I.2: given a point A and a segment BC , construct an equal segment starting at A .
- i. Euclid used the word *equal* the way we use *congruent* now.
 - ii. You can do it by simply opening a standard compass, setting one end on A , and the other anywhere, at a point I'll call K . Euclid couldn't do that, because his compass wouldn't stay open. Moreover, he wanted his proof to follow from his postulates, and they say nothing about a compass. He had to use his postulates and proposition I.1 to find a suitable point K .
 - (1) Use postulate (c) to construct the circle \mathfrak{B} with center B and radius BC .
 - (2) Use proposition I.1 to construct an equilateral $\triangle ABD$.
 - (3) Use postulate (b) to extend segment DB past B to intersect \mathfrak{B} . Call that intersection G .
 - (4) Use postulate (c) to construct the circle \mathfrak{D} with center D and radius DG .
 - (5) Use postulate (b) to extend segment DA past A to intersect \mathfrak{D} . Call that intersection K .
 - (6) Now show that segments AK and BC are equal.
 - iii. Postulate (b) justifies speaking of the intersections G and K only if "continuously" is construed the way I just used the postulate. Euclid didn't say it that way, and there's no good evidence that he meant to do that. Modern, "cleaned-up" versions of Euclid often take the existence of such an intersection as a separate, explicit, postulate.
- c. Proposition I.3: to cut off from a given segment AB one equal to a given segment r shorter than AB .
- i. Mr. Wong presented Euclid's proof: use proposition I.2 to construct a segment BC equal to r , then postulate (c) to construct the circle with center B and radius BC ; its intersection with AB is the desired cutting point.
 - ii. Again, Euclid's postulates don't justify the existence of the intersection. Modern versions use a postulate that does so explicitly.
- d. Proposition I.4 (SAS—side-angle-side): if triangles have two pairs of equal edges, and the angles between each pair are equal, then the remaining pairs of edges and angles are equal.

- i. I presented Euclid's proof, essentially as follows. Given triangles ABC and DEF such that $\angle A = \angle D$, $AB = DE$, and $AC = DF$,
- ii. E is the only point on ray DE such that lengths AB, DE are equal. I suppose this is "justified" by the proposition about cutting off a segment equal to a given segment. But that proposition doesn't say explicitly that there is only one possibility.
- iii. DF is the only ray on the F side of line DE such that $\angle A = \angle EDF$. Is there a proposition in Euclid about constructing such an angle? If so, that would "justify" this step as in (ii); but he hasn't done so here.
- iv. F is the only point on ray DF such that lengths AC, DF are equal. This would be "justified" as in (ii).
- v. There is only one segment between E and F . That's a postulate.
- vi. Thus, by "applying" ABC to DEF , distances BC, EF are seen to be equal. He went on to conclude that the remaining pairs of angles are equal. "Application" is sometimes called "superposition." But Euclid didn't say anything about it in the postulates. Maybe he was taking it as an undefined notion, maybe he left unstated some postulates about it. We don't know what he intended.
- vii. Modern authors generally take SAS as an additional postulate.