Mathematics Department Colloquium
San Francisco State University

6 October 2010

Young Tarski

James T. Smith, Professor Emeritus

Joint work with Mr. and Mrs. Andrew and Joanna McFarland, of Płock, Poland.

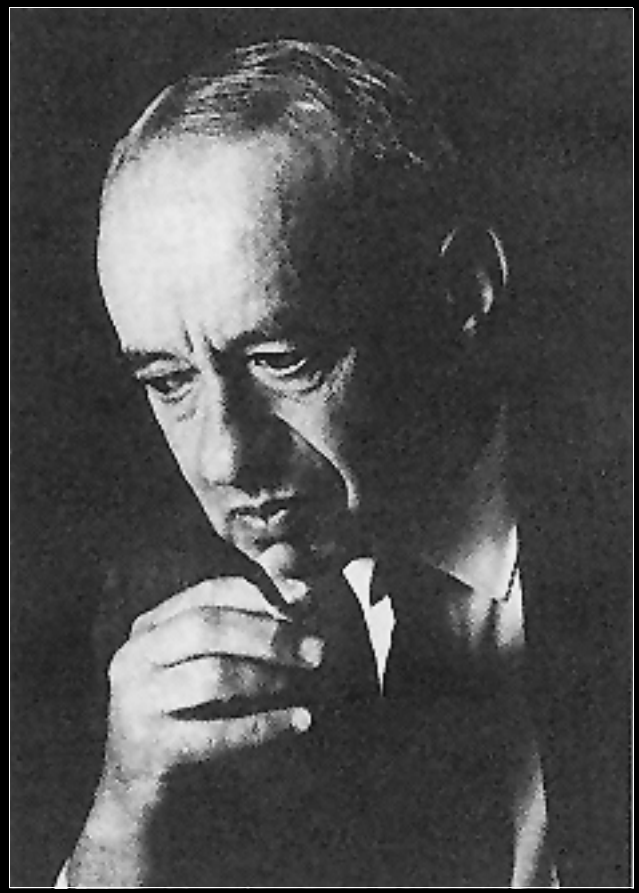
I thank the CSU retirement system for making this study possible.
Ms. Ula Furman has assisted.

Young Tarski

James T. Smith

- **Who was Alfred Tarski?**
  - **1901–1939, Warsaw; 1942–1983, Berkeley.**
  - **1923–1953: perfected our framework for research in mathematical logic.**
  - **1953–1983: its preeminent figure.**
  - **My teacher's teacher, my external PhD examiner.**

- **Biography**
  - **Feferman & Feferman 2004**



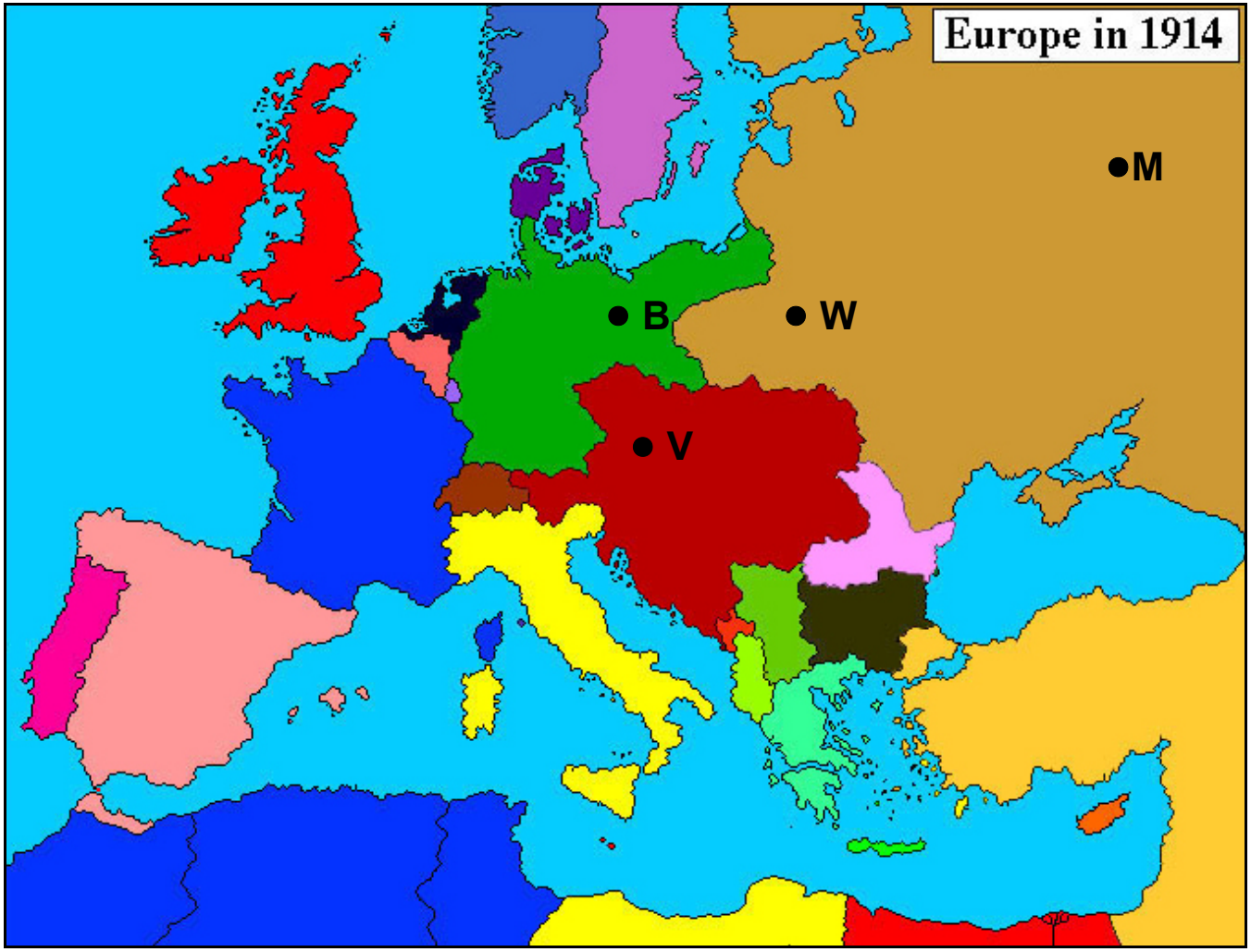
Alfred Tarski
1901–1983

Young Tarski

- **Goals of this presentation**
  - **Describe Tarski's upbringing and early career in Poland,**
    - **emphasizing the extreme social turmoil, and**
    - **contrasting his environment with yours.**
  - **Depict Tarski's dual roles in geometry**
    - **as mathematical researcher and schoolteacher.**
  - **Note where needed historical work is underway.**

Young Tarski: Family & School (1)

- **Born in 1901 in Warsaw, Poland,**
  - **at that time in the Russian empire.**
  - **Poland had long been partitioned between Germany, Russia, Austria.**



M = Moscow

B = Berlin
W = Warsaw

V = Vienna

Young Tarski: Family & School (2)

- **Warsaw 1901: about 685K, 30% Jewish.**
  - **SF 2008: about 800K, 31% Asian background.**
- **He was Alfred *Teitelbaum*, a Jew.**
- **Father and mother stemmed from prosperous families,**
  - **in the lumber and textile businesses.**

Rosa

Ignacy

Alfred

- **The Teitelbaums lived near the city center in a neighborhood now mostly reconfigured,**
  - **but their comfortable apartment building, remodeled, still stands.**
- **Secular and assimilated,**
  - **they spoke Polish at home.**
  - **(Most Warsaw Jews spoke Yiddish at home, not Polish.)**

Young Tarski: Family & School (4)

- **At 9, Alfred entered a Russian public school.**
  - **He also studied French, German, Greek, and Latin,**
  - **attended temple to study Hebrew and the Torah,**
  - **and had private lessons in Polish.**
- **At 11 he translated a (rather dark) German story as a gift to his parents.**
- **In maturity he complained bitterly about childhood antisemitic hazing.**

How did Alfred's childhood differ from yours?

Now we switch to another strand of this story.

Young Tarski: Nation & High School (1)

- Recall the longstanding partition of Poland.
  - Thoughts of unification, independence → agitation, intrigue.
- War broke out in August 1914 for other reasons.
- Nearly 2M Poles served,
  - most under Józef Piłsudski,
  - with Austria & Germany against Russia.
- Britain, France: Poland's a Russian problem.
- Piłsudski played the sides ⇨⇦ :
  - more posturing, intrigue.
- Alfred became a Polish nationalist.

Young Tarski: Nation & High School (2)

- **January 1915: Alfred → small, elite Szkoła Mazowiecka.**
  - **Faculty highly educated, e.g.**

    biology: Stanisław Przyłęcki, MD Geneva

    philosophy: Stefan Frycz, PhD Lwów

    Both became profs. at Warsaw.
- **August 1915: Germans took Warsaw.**
  - **Instruction → Polish.**
- **Background, February 1917—March 1918:**
  - **civil war in Russia, collapse of the empire.**

Young Tarski: Nation & High School (3)

- **Summer 1918:**
  - **2 classmates too ill to finish (flu??),**
  - **1 flunked,**
  - **Alfred & 6 others were graduated.**
- ***Excellent!* on all his examinations:**
  - **Polish, Latin, German, French,**
  - **Literature, Mathematics,**
  - **Physics, Cosmography, Hygiene.**

How did Alfred's high-school experience differ from yours?

Young Tarski: Nation & High School (4)

- **The U. of Warsaw had served only the Russian establishment.**
  - **Polish students had to study abroad.**
- **Fall 1915: Germans fostered it as a new Polish university,**
  - **with a new Polish faculty.**
  - **Topologist Kazimierz Kuratowski, then a student:**

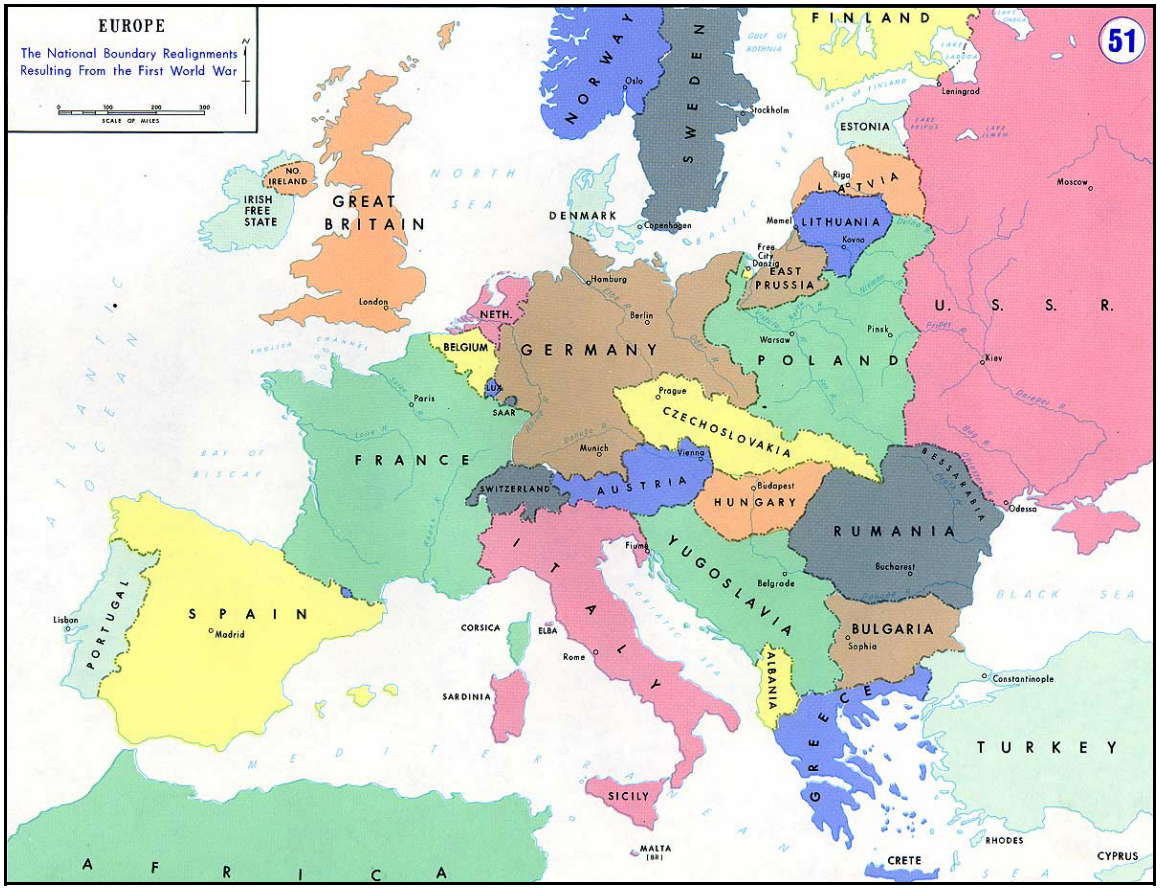
    …the [Warsaw] atmosphere…released a great creative potential…which produced a surprising development in many branches of science…. (1980, 28)
- **Fall 1918: Alfred entered University to study biology:**
  - **full load of science courses.**
  - **But it closed due to war's-end chaos.**
  - **We don't know exactly why, nor what Alfred did during 1918–1919.**

- **New Poland!**

  New Government

  New University

  New Mathematics



- **Kuratowski:**

One of the principal means…for [realizing the University's potential] was the concentration...in a relatively narrow field of mathematics…one in which Polish mathematicians had common interests and…achievements which counted on a world scale. This field comprised set theory together with topology, and the foundations of mathematics together with mathematical logic. (1980, 31)

Young Tarski: First University Year (1)

- **Feferman & Feferman:**

  [Alfred's] social identity had been transformed from that of a moderately assimilated bourgeois Jewish boy to that of a Polish patriot. He was neither religious nor a Zionist; politically his leanings were socialist. (2004, 16)

- **The atmosphere was *electric*. Historian Andrzej Garlicki:**

  …students could be met more often at political rallies and meetings than in university lecture rooms or laboratories. (1982, 341)

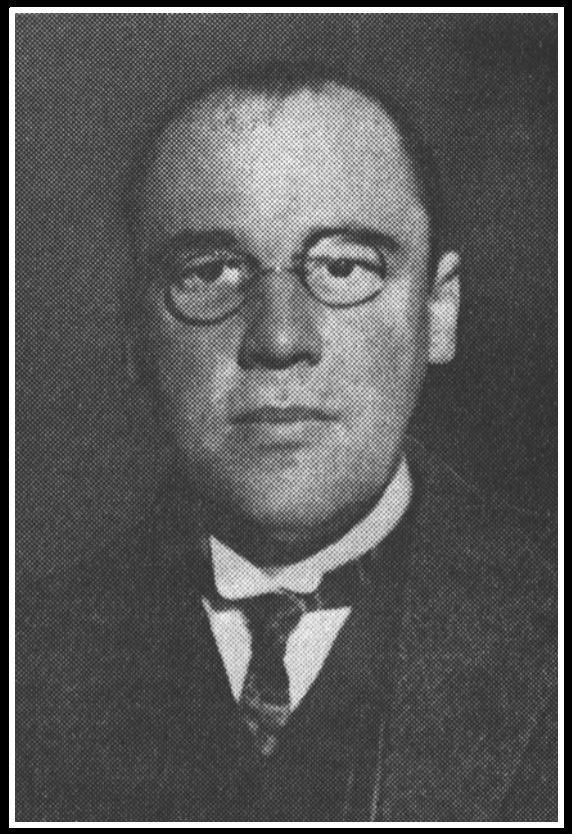
- **Alfred learned about the excitement in mathematics, too.**

- **Probabilist Mark Kac, later a student in Lwów:**

  Polish universities have always been graduate or professional schools. There was nothing corresponding to the American undergraduate school. [Students had to acquire that maturity in high school.] (1985, 26)

Young Tarski: First University Year (2)

- **Alfred reenrolled for 1919–1920 to study math, logic:**
  - **physics, sociology**
  - **Tadeusz Kotarbiński, logic & philosophy**
  - **Stefan Mazurkiewicz, calculus**
  - **Wacław Sierpiński, set theory & measure theory**
  - **Stanisław Leśniewski, set theory & foundations seminar**

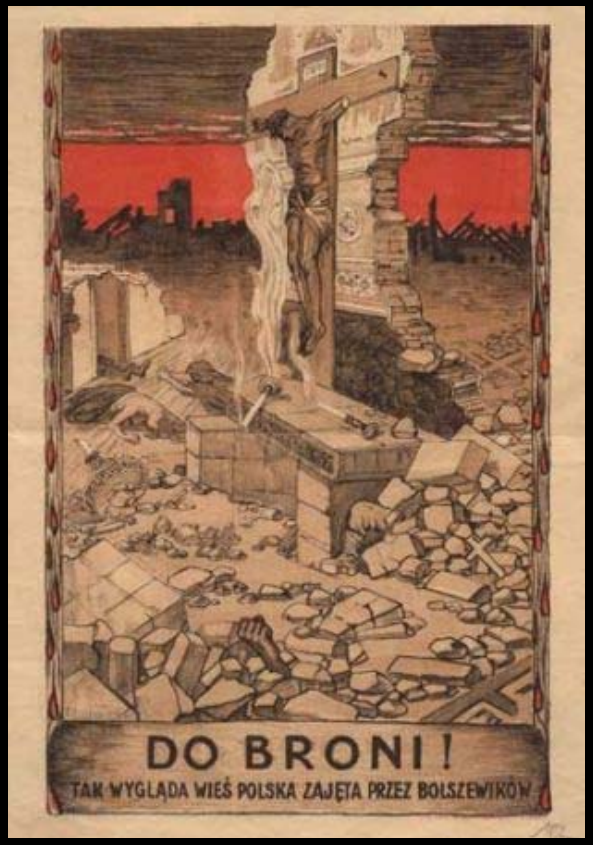


Wacław Sierpiński
1882–1969

Stanisław Leśniewski
1886–1939

Young Tarski: First University Year (4)

- **Strife, chaos: Poland fought 6 wars 1919–1922.**
  - **Math. profs. worked on decoding Russian messages.**
- **Spring 1920**
  - **1 M Poles under Piłsudski ⇨⇦ Bolsheviks under Lenin, Trotsky.**
  - **Nationalism, antisemitism rampant.**



To Arms!

This is what a village occupied by Bolsheviks looked like.

Bolshevik freedom

Give you room
Give you freedom
Give you the land
Work and bread

Basely cheated
Made war with Poland
Instead of freedom—the fist
Instead of land—requisition
Instead of working—misery
Instead of bread—hunger

Hey!

Whoever is a Pole
To your Bayonets!!

Young Tarski: First University Year (6)

- **Summer 1920**
  - **Russians → Warsaw outskirts. Garlicki:**

    In July, the students again adopted a resolution to join the army en masse, and for the second time lectures were suspended. (1982, 341)
  - **Officially unfit,**
    - **Alfred volunteered for a supply and medical unit.**
    - **Research question: were Jews kept away from the front?**
  - **Decoding → total defeat of Russians.**
- **Struggle for independence would soon be over!**

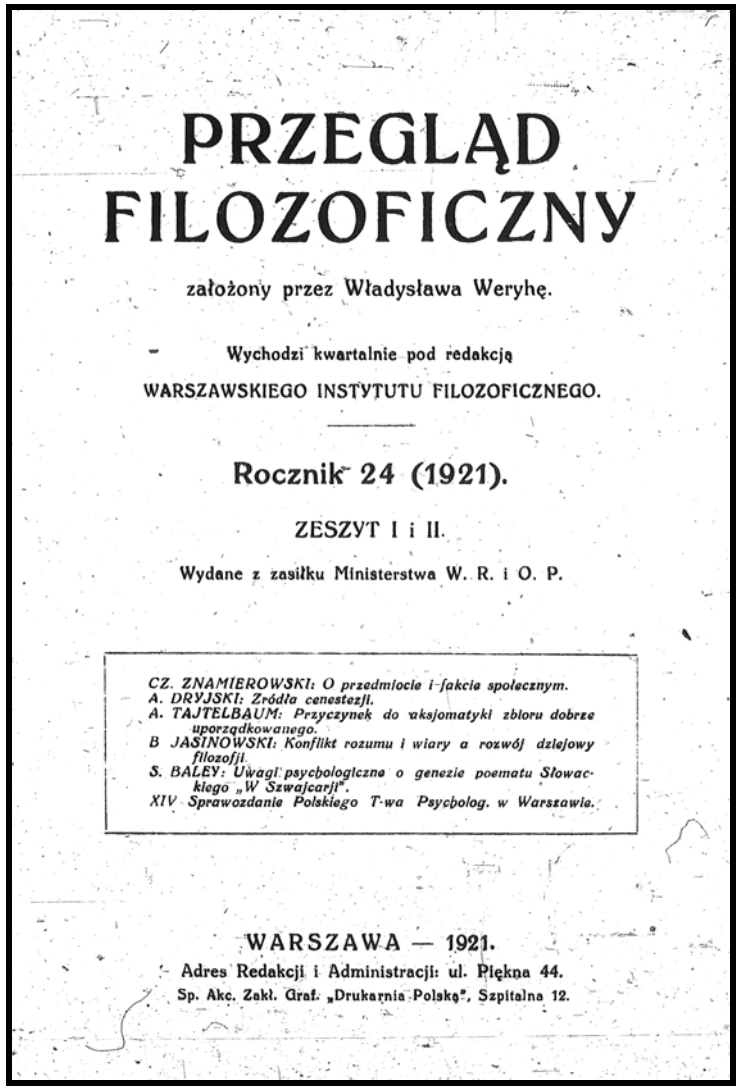
How did Alfred's first university year differ from yours?

Young Tarski: On the Runway (1)

- **Fall 1920, Alfred returned to classes:**
  - **Mazurkiewicz, analysis**
  - **Sierpiński, algebra, set theory**
  - **Jan Łukasiewicz, logic**
- **Changed signature to *Tajtelbaum*.**
- **1st year work in Leśniewski's seminar**
  - → **1st research paper (1921):**
  - **Contribution to the axiomatics of well-ordered sets**
  - **Main result:**

    a relation R on a set $Z \neq \phi$ is a well-ordering

    $\leftrightarrow (\forall U \subseteq Z)\big(\phi \neq U \Rightarrow (\exists! a \in U)(\forall u \in U) \neg u R a\big)$.
  - **Today an exercise in a graduate course.**



PRZEGLĄD FILOZOFICZNY

założony przez Władysława Weryhę.

Wychodzi kwartalnie pod redakcją
WARSZAWSKIEGO INSTYTUTU FILOZOFICZNEGO.

Rocznik 24 (1921).

ZESZYT I i II.

Wydane z zasiłku Ministerstwa W. R. i O. P.

CZ. ZNAMIEROWSKI: O przedmiocie i fakcie społecznym.
A. DRYJSKI: Źródła cenestezji.
A. TAJTELBAUM: Przyczynek do aksjomatyki zbioru dobrze uporządkowanego.
B JASINOWSKI: Konflikt rozumu i wiary a rozwój dziejowy filozofji.
S. BALEY: Uwagi psychologiczne o genezie poematu Słowackiego „W Szwajcarji".
XIV Sprawozdanie Polskiego T-wa Psycholog. w Warszawie.

WARSZAWA — 1921.
Adres Redakcji i Administracji: ul. Piękna 44.
Sp. Akc. Zakł. Graf. „Drukarnia Polska", Szpitalna 12.

Young Tarski: On the Runway (2)

- **Journal editors: Kazimierz Twardowski (at Lwów), Marjan Borowski.**
- **Spring and Fall 1921**
  - **Talks at Warsaw Philosophical Institute extending ideas of Leśniewski, Kazimierz Ajdukiewicz.**
- **Students of Twardowski:**
  - **Ajdukiewicz, Borowski, Frysz, Kotarbiński, Leśniewski, Łukasiewicz.**
- **Leśniewski obsessively precise, sharply critical of others, who did not attain that standard.**
- **Borowski to Twardowski:**

  …I inform you—discreetly—that *Przegląd filozoficzny* doesn't have many papers…in the editorial office. Warsaw coryphaei write little, being afraid of Leśniewski!—Although, the "scourge of God" has also risen upon him, in the person of his pupil, Tajtelbaum. (1922)

Young Tarski: Taking Off (1)

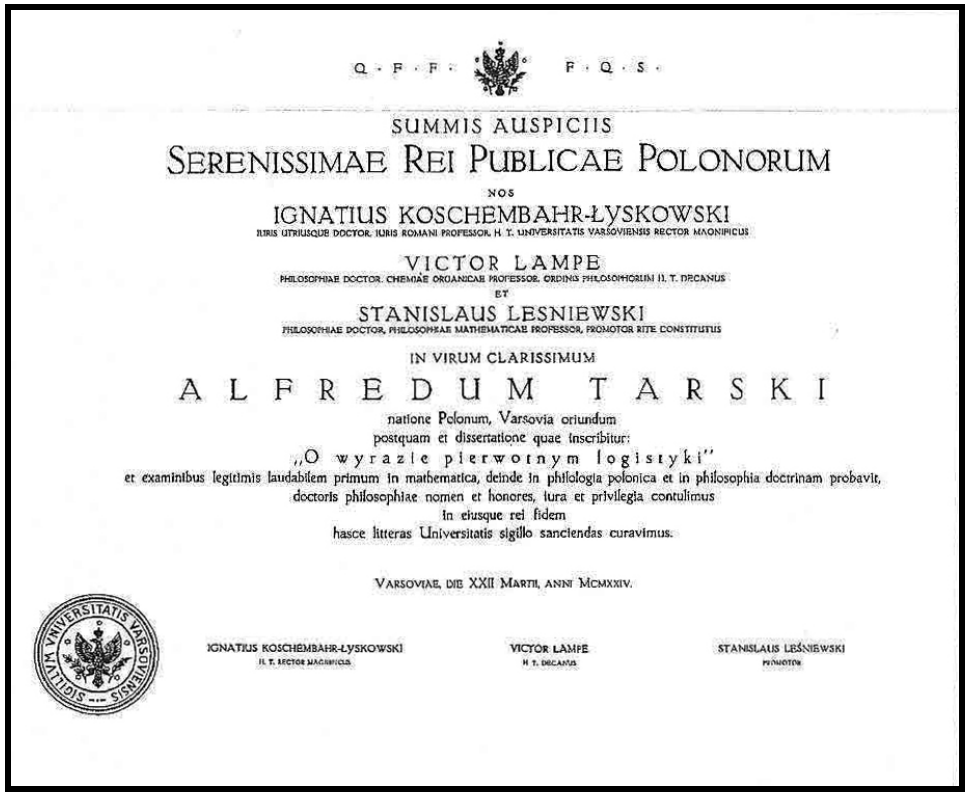
- **1920–1922**
  - **Taught geometry at a girls' high school.**
  - **Fired for being Jewish.**
- **1922–1925**
  - **Gave logic courses at National Pedagogical Institute.**
  - **Taught math. at a *Jewish* girls' high school.**
- **Kac on Jews' academic employment:**

  …all doubts vanished if the candidate were to convert to Catholicism. Polish anti-Semitism had always been largely religious. (1985, 28)

  ***I doubt that, but that's what* he *thought*.**
- **Spring 1922: Converted to Catholicism.**
- **Fall 1923: Changed name to *Tarski*.**

Young Tarski: Taking Off (2)

- **Spring 1924**
  - **PhD in logic under Leśniewski**
  - ***On the primitive term of logistic***
- **1925–1939**
  - **Taught geometry at the Stefan Żeromski school.**
- **He now had a secure position,**
  - **but not at a university.**



Q · F · F · F · Q · S ·

SUMMIS AUSPICIIS

SERENISSIMAE REI PUBLICAE POLONORUM

NOS

IGNATIUS KOSCHEMBAHR-ŁYSKOWSKI

IURIS UTRIUSQUE DOCTOR, IURIS ROMANI PROFESSOR, H. T. UNIVERSITATIS VARSOVIENSIS RECTOR MAGNIFICUS

VICTOR LAMPE

PHILOSOPHIAE DOCTOR, CHEMIAE ORGANICAE PROFESSOR, ORDINIS PHILOSOPHORUM H. T. DECANUS

ET

STANISLAUS LESNIEWSKI

PHILOSOPHIAE DOCTOR, PHILOSOPHIAE MATHEMATICAE PROFESSOR, PROMOTOR RITE CONSTITUTUS

IN VIRUM CLARISSIMUM

A L F R E D U M T A R S K I

natione Polonum, Varsovia oriundum

postquam et dissertatione quae inscribitur:

„O wyrazie pierwotnym logistyki"

et examinibus legitimis laudabilem primum in mathematica, deinde in philologia polonica et in philosophia doctrinam probavit,

doctoris philosophiae nomen et honores, iura et privilegia contulimus

In eiusque rei fidem

hasce litteras Universitatis sigillo sanciendas curavimus.

VARSOVIAE, DIE XXII MARTII, ANNI MCMXXIV.

SIGILLUM UNIVERSITATIS VARSOVIENSIS

IGNATIUS KOSCHEMBAHR-ŁYSKOWSKI
H. T. RECTOR MAGNIFICUS

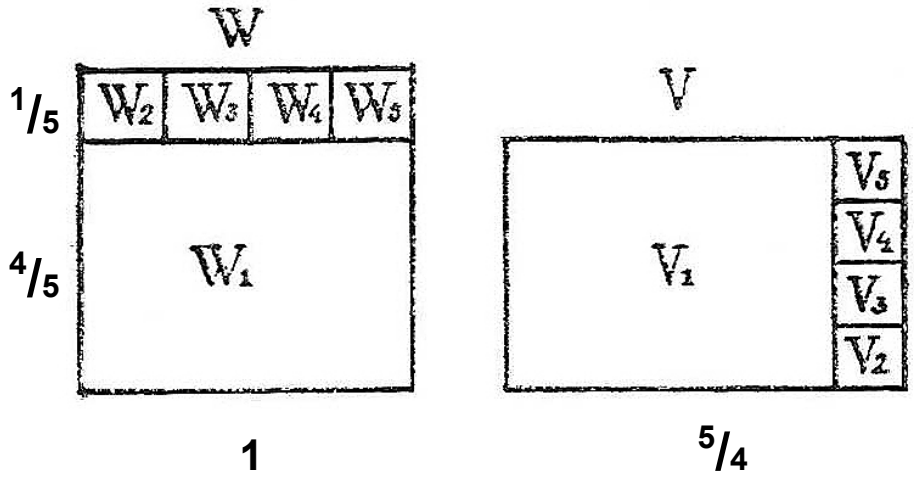
VICTOR LAMPE
H. T. DECANUS

STANISLAUS LEŚNIEWSKI
PROMOTOR

How did Tarski's experiences differ from yours, or from those of mathematicians you know?

Young Tarski: Geometry (1)

- **5 long threads already emerging:**
  - **set theory, measure theory, teacher prep., geometry, logic.**
  - ***Concentrate on some facets of first 4*.**
- **Preparing teachers, Tarski must have been thinking about this:**
  - **Polygons W, V have = area ↔ *dissectible* into**
    = finite numbers of ≅ polygons with disjoint interiors.



(Tarski 1931, fig 1)

- **For polyhedra, the same is not true.**
- **Reference: Sally & Sally, 2007, chapter 5.**

Young Tarski: Geometry (2)

- **Powerful tools: Sierpiński's courses, Hausdorff 1914.**
- **Spring 1923, Lwów, 1st Polish Philosophy Congress:**
  - **secretary of the logic section, Alfred**
  - **met Stefan Banach, soon famous in analysis.**
- **They collaborated:**
  - **Banach 1923, *On the problem of measure*:**

    $\exists$ additive $\cong$-invariant extension of area to all bounded subsets of $\mathbb{R}^2$ (but not of volume in $\mathbb{R}^3$).

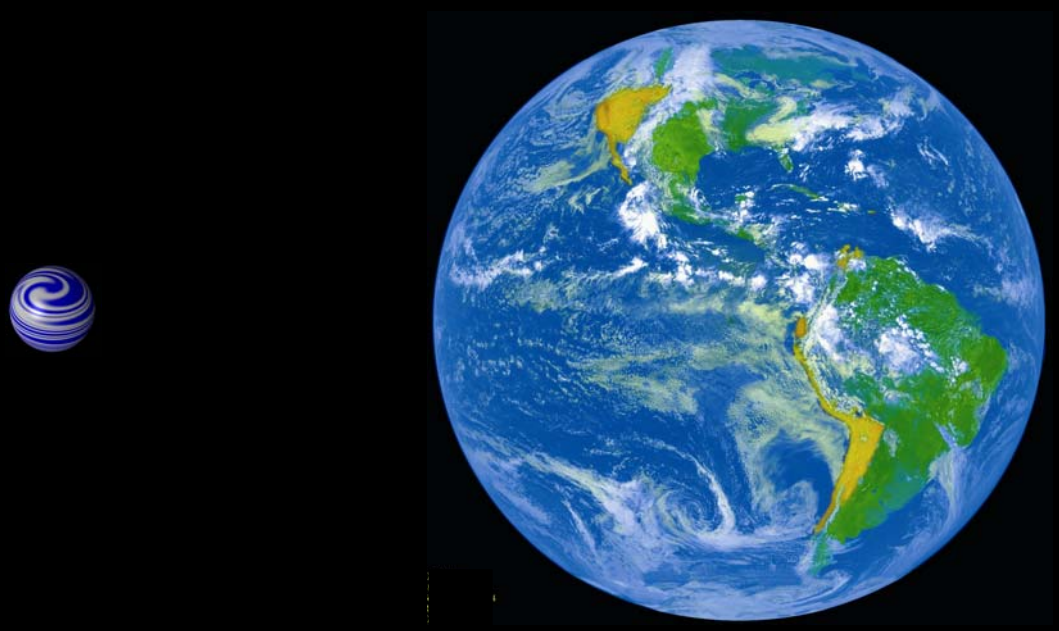
  - **Tarski 1924, *On the equivalence of polygons*:**

    Polygons have = area $\leftrightarrow$ *decomposible* into = finite numbers of $\cong$ disjoint sets.

    (Uses Banach 1923.)

Stefan Banach
1892–1945

- **Banach–Tarski 1924, *On decomposition of point sets into respectively congruent parts*:**
  - **In $\mathbb{R}^3$ any 2 bounded sets with interior are decomposible into = finite numbers of disjoint $\cong$ sets.**



Banach–Tarski Paradox

Set-theoretic decomposition doesn't correspond with physical or intuitive decomposition.

Young Tarski: Teaching Geometry (1)

Tarski → postdoc → lecturer, but never a professor in Poland,

↓ although known worldwide in logic.

↘

high-school teacher

Young Tarski: Teaching Geometry (2)

- **Two 1931–1932 papers & one by Henryk Moese**
  - **in *Young Mathematician*, a journal for teachers and students,**
  - **on σ(W,V), the degree of equivalence of 2 polygons with = area: smallest number of ≅ pieces required.**

ROK 1 — MARZEC 1931 — NR. 3

MŁODY MATEMATYK

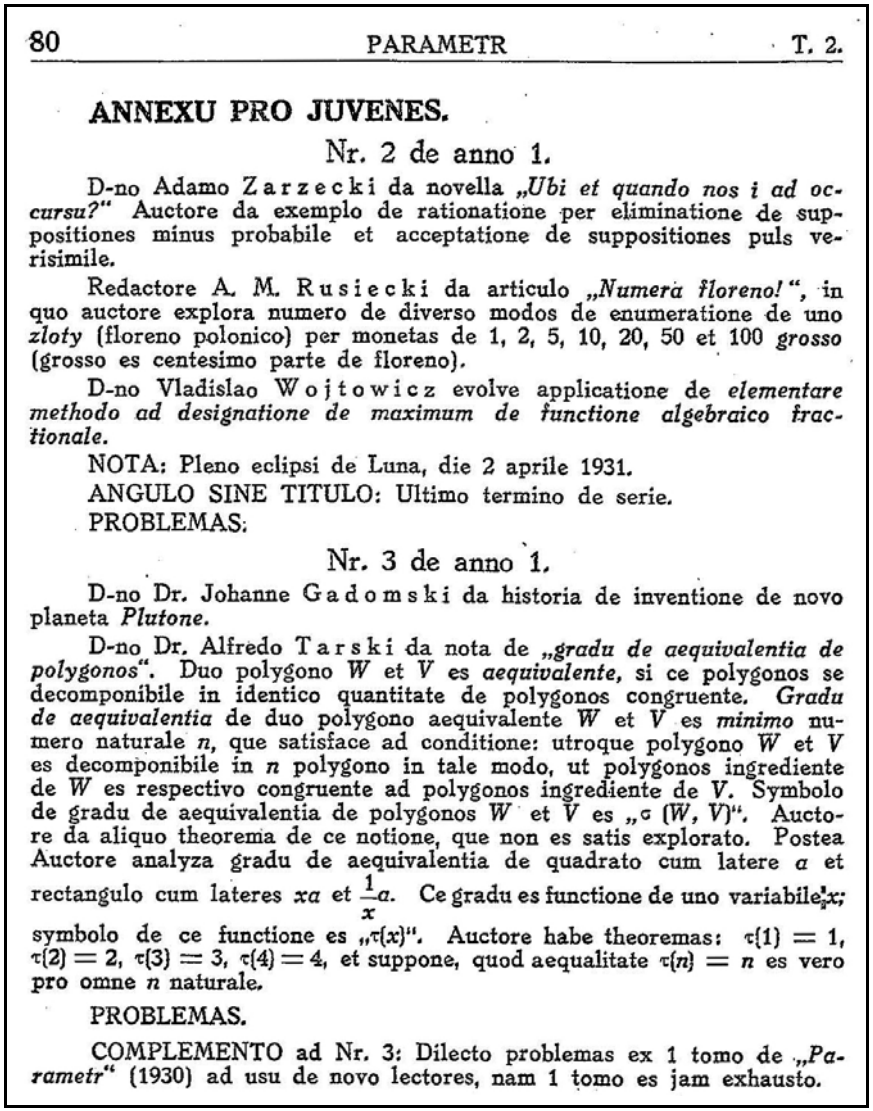
CZASOPISMO DLA MŁODZIEŻY SZKOLNEJ
WYCHODZI POD REDAKCJĄ A. M. RUSIECKIEGO
PRZY WSPÓŁUDZIALE S. STRASZEWICZA.

TREŚĆ: J. Gadomski. Odkrycie Plutona. — A. Tarski. O stopniu równoważności wielokątów. — Rozwiązania zadań Nr. 40 i 57. — Zadania Nr. 167 — 181.

DR. JAN GADOMSKI (Warszawa).

Odkrycie Plutona.

Jednem z najciekawszych dla miłośników nieba wydarzeń astronomicznych w ciągu lat ostatnich jest odkrycie nowej wiątej „wielkiej" planety systemu słonecznego. Okoli... rzyszące jej odszu...



80 PARAMETR T. 2.

ANNEXU PRO JUVENES.

Nr. 2 de anno 1.

D-no Adamo Zarzecki da novella *„Ubi et quando nos i ad occursu?"* Auctore da exemplo de rationatione per eliminatione de suppositiones minus probabile et acceptatione de suppositiones puls verisimile.

Redactore A. M. Rusiecki da articulo *„Numera floreno!"*, in quo auctore explora numero de diverso modos de enumeratione de uno *zloty* (floreno polonico) per monetas de 1, 2, 5, 10, 20, 50 et 100 *grosso* (grosso es centesimo parte de floreno).

D-no Vladislao Wojtowicz evolve applicatione de *elementare methodo ad designatione de maximum de functione algebraico fractionale.*

NOTA: Pleno eclipsi de Luna, die 2 aprile 1931.

ANGULO SINE TITULO: Ultimo termino de serie.

PROBLEMAS.

Nr. 3 de anno 1.

D-no Dr. Johanne Gadomski da historia de inventione de novo planeta *Plutone.*

D-no Dr. Alfredo Tarski da nota de *„gradu de aequivalentia de polygonos"*. Duo polygono W et V es *aequivalente*, si ce polygonos se decomponibile in identico quantitate de polygonos congruente. *Gradu de aequivalentia* de duo polygono aequivalente W et V es *minimo* numero naturale n, que satisface ad conditione: utroque polygono W et V es decomponibile in n polygono in tale modo, ut polygonos ingrediente de W es respectivo congruente ad polygonos ingrediente de V. Symbolo de gradu de aequivalentia de polygonos W et V es „$\sigma(W, V)$". Auctore da aliquo theorema de ce notione, que non es satis explorato. Postea Auctore analyza gradu de aequivalentia de quadrato cum latere a et rectangulo cum lateres xa et $\frac{1}{x}a$. Ce gradu es functione de uno variabile x; symbolo de ce functione es „$\tau(x)$". Auctore habe theoremas: $\tau\{1\} = 1$, $\tau\{2\} = 2$, $\tau\{3\} = 3$, $\tau\{4\} = 4$, et suppone, quod aequalitate $\tau\{n\} = n$ es vero pro omne n naturale.

PROBLEMAS.

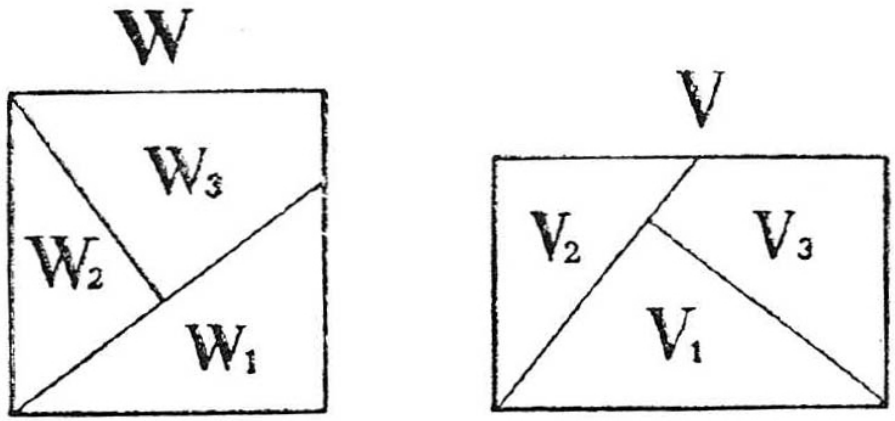
COMPLEMENTO ad Nr. 3: Dilecto problemas ex 1 tomo de *„Parametr"* (1930) ad usu de novo lectores, nam 1 tomo es jam exhausto.

Contents in *Lingua Peano!*

Young Tarski: Teaching Geometry (3)

- **Previous example: W, V with 5 parts**
  - **Look: $\sigma(W,V) \leq 3$.**
  - **Can you cut them to prove**

    $\sigma(W,V) = 2$?

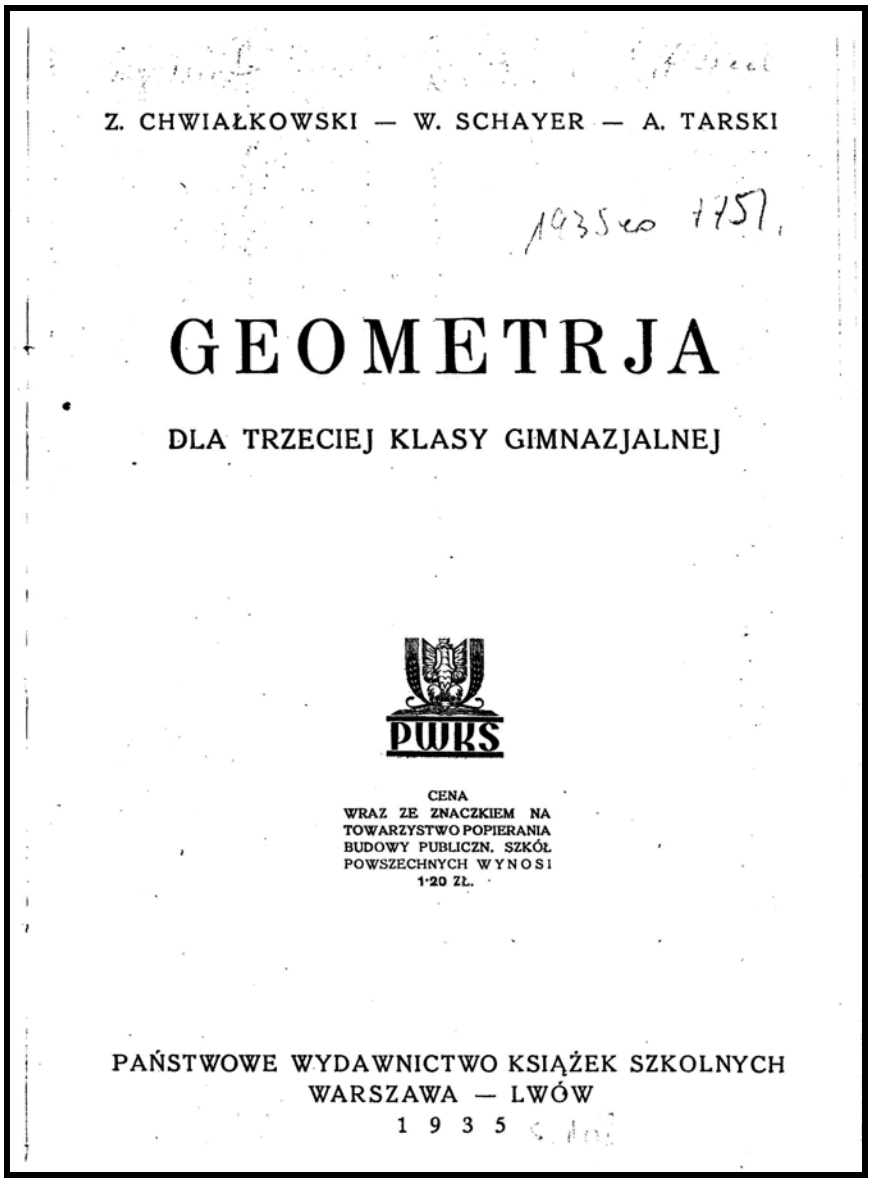


(Tarski 1931, fig 2)

- **Tarski invited others to pursue this subject,**
  - **but that never happened.**

Young Tarski: Teaching Geometry (4)

- **1935**
  - **Coauthored a high-school text.**
  - **We know little about the others.**
  - **Nor how it was used.**
  - **Last chapters, on area.**
  - ***We're working on it!***



Z. CHWIAŁKOWSKI — W. SCHAYER — A. TARSKI

1935 co 1757.

GEOMETRJA

DLA TRZECIEJ KLASY GIMNAZJALNEJ

PWRS

CENA
WRAZ ZE ZNACZKIEM NA
TOWARZYSTWO POPIERANIA
BUDOWY PUBLICZN. SZKÓŁ
POWSZECHNYCH WYNOSI
1·20 ZŁ.

PAŃSTWOWE WYDAWNICTWO KSIĄŻEK SZKOLNYCH
WARSZAWA — LWÓW
1 9 3 5

Young Tarski: Teaching Geometry (5)

- **Jaroslaw Rudniański remembered Tarski from the 1930s:**
  - **controlled but a bit nervous…,**
  - **cheerful but with a "clouded" face…,**
  - **patience itself while explaining…, and**
  - **loved by the gifted. (Jadacki 2003a, 145)**

- **Witold Kozlowski told us**
  - **the gifted would visit Tarski's home.**
  - **Tarski's Univ. students → his school lectures.**
  - **Tarski's favorite subject: area measure.**

Alfred Tarski

Joanna & Andrew McFarland

Witold Kozlowski (1919–)

Tarski: Conclusion

- Tarski, on a lecture trip, was stranded in the U.S. when the Nazis invaded Poland.
- Nazis murdered his entire extended family,
  - except his wife (not a Jew) and their son and daughter.
- Tarski became prof. at Berkeley in 1943.
  - He made Berkeley the world's center for logic.
- He presented his favorite subject—degree of equivalence—to Berkeley schoolkids,
  - and after my thesis defense in Regina in 1970.
- The 1923–1924 Banach & Tarski papers → much new mathematics:
  - see Sally & Sally 2007, chapter 5.
- Tarski's pubs. mentioned here will be translated in an edition by the McFarlands and me.

Mathematics Department Colloquium
San Francisco State University

6 October 2010

Thank you for your interest!

James T. Smith, Professor Emeritus

Young Tarski: References

James T. Smith

Biography Feferman & Feferman 2004, 2009; Givant 1991, 1999; Jadacki 2003a.
Geometry Sally & Sally 2007, chapter 5; Wagon 1985.
History Davies 1982

Banach, Stefan. 1923. Sur le problème de la mesure. *Fundamenta Mathematicae* 4: 7–33.

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Illustrations

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