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Thomsen's equation.

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Let a , b , and c be the lines opposite corresponding vertices of a triangle $\triangle ABC$. The reflections across the lines a , b , and c are three involutions S_a , S_b , and S_c in the Euclidean plane, i.e., $S_a^2 = S_b^2 = S_c^2 = i$ is the identity motion. Then the following equation with twenty-two reflections on the left holds for every triangle:

$$S_a S_b S_c S_a S_b S_c S_b S_c S_a S_b S_c S_a S_c S_b S_a S_c S_b S_c S_b S_a S_c S_b = i.$$

The above equation is called Thomsen's equation. The author proves that Thomsen's equation is the shortest equation which cannot be derived from $S_a^2 = S_b^2 = S_c^2 = i$. He shows that there are no corresponding results for higher dimensions. The author's historical and bibliographical notes make the paper very interesting.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.