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Pieri, Tarski, and Axiomatics

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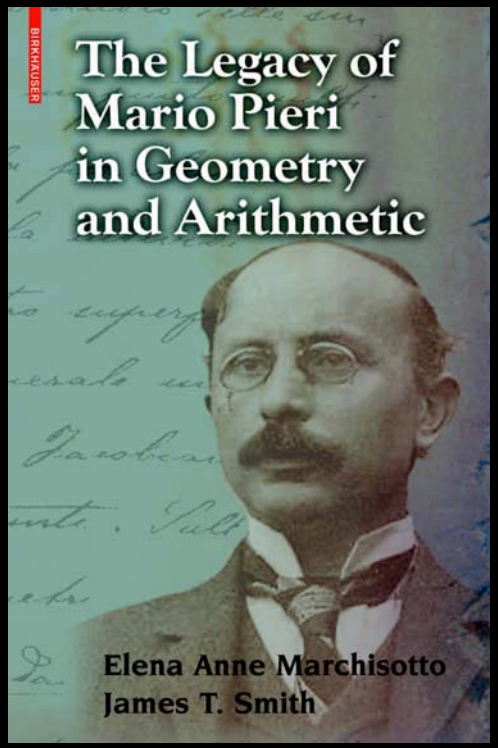
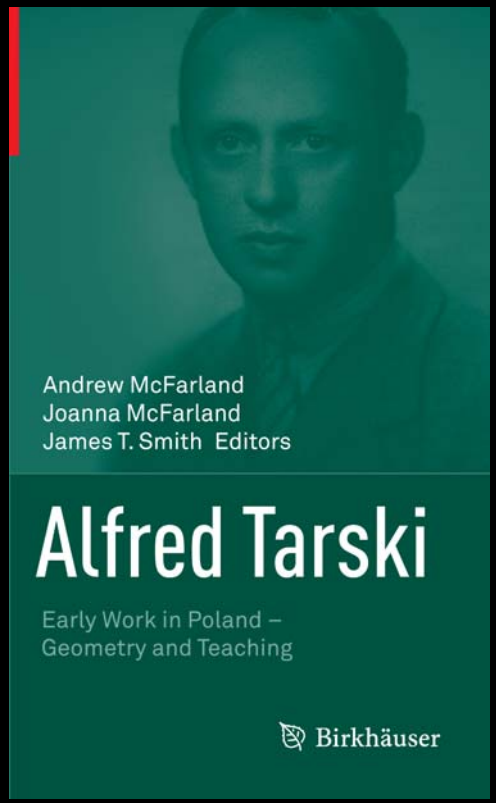
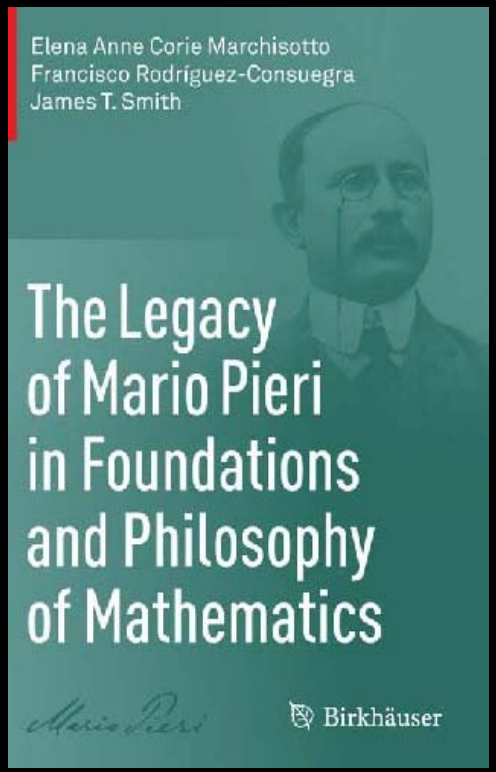






2007 **2012** **2021**

- **Question after Fred Rickey's 21 October presentation:**

  Are any specific contributions of Leśniewski discernible in typical mathematical-logic textbooks?

  I think Fred answered *No*.

- **How about Pieri's and Tarski's contributions? *YES!***

  Tarski's are famous.

  Pieri's?

- **My presentation traces two themes in axiomatics:**
  - ***Selecting primitive concepts and formulating postulates* sufficient to develop a familiar informal theory;**
  - **Deducing its theorems as their *logical consequences*.**

  I'll concentrate now on the second.

- **This is Whig history: how the past led to the present.**

Modern Definition of *Logical Consequence*

- ***Logical consequences*** of a family A of sentences are sentences that are true in all models of A.
- ***Sentences?*** Those in some fragment L of mathematical language.
- ***Models of A?*** Structures for L in which all sentences of A are true.
- ***Structure for L?*** A set $U \neq \varnothing$ and some relations over U that L mentions.

I'm restricting the language features, and assuming we know what *true* means.

- A sentence is ***logically independent*** of A $\Leftrightarrow$ if it is *not* a logical consequence.

Find extreme detail in Epstein 2006: *Classical Mathematical Logic*, ch. III–IV (46 pp).
Concise in Shoenfield 1957: *Mathematical Logic*, §§2.5–2.6 (6 pp). (I learned here.)
Source: Tarski 1953, *A General Method in Proofs of Undecidability*, p. 8.

What about **Tarski [1935] 1936**, *Über den Begriff der logischen Folgerung*? There, Tarski didn't mention *structures*: variables should range over *all* objects. There, $(\exists x,y)\ x \neq y$ was a consequence of the family $A = \varnothing$: it was ***logically true***. With the modern definition, it is not. Tarski didn't write about this discrepancy, nor did anyone else until after his death. But Tarski [1935] 1936 was a milestone. **How did he arrive there?**

Young Tarski

- In Warsaw, a city in Russia in 1901
  —about 685K, 30% Jewish—
  he was born Alfred Teitelbaum, a Jew.
- Stemming from mercantile families, he lived comfortably near the city center.
- Secular and assimilated, his family of 4 spoke Polish at home (not Yiddish).
- His schooling was in Russian.
- In August 1915, the German army took Warsaw. His high school became Polish.
- [By his 1918 graduation] his social identity had been transformed from that of a moderately assimilated bourgeois Jewish boy to that of a Polish patriot. He was neither religious nor a Zionist; politically his leanings were socialist. (Feferman & Feferman 2004, 16)
- In 1915 the University became Polish, and began rapid expansion and modernization.

51 Koszykowa Street
(2012, renovated)

Tarski's Debut and Livelihood

- Alfred studied *set theory* with mathematicians Stanisław Mazurkiewicz and Wacław Sierpiński; and *logic* with philosophers Tadeusz Kotarbiński, Stanisław Leśniewski, and Jan Łukasiewicz (former Lwów students of Kazimierz Twardowski).
- Another Twardowski graduate, *Kazimierz Ajdukiewicz*, visited Warsaw at this time. His [1921]1966 *Habilitationsschrift*, a survey of methods now basic to elementary logic, was the subject of Tarski's first public presentation, in 1921.

Kazimierz Ajdukiewicz
around 1920

- Alfred finished PhD study in 1924, with Leśniewski. (Recall Fred Rickey's talk.)
- That year, Tarski published several major papers, including Banach & Tarski 1924.
- He changed his surname to Tarski (sounds Polish, not Jewish).
- Tarski needed a job. After their great expansion, Polish universities had "hired up". Tarski could only work there part-time, as an assistant. He taught teacher-education courses and led the logic-research seminar until 1939, earning world recognition as a researcher.
- Tarski worked full-time as a high-school teacher.

Tarski, Ajdukiewicz, and Logical Consequence

- During 1926–1928 Tarski and Ajdukiewicz taught geometry and philosophy at the same Warsaw high school.
- Ajdukiewicz [1921] 1966 had included a discussion of logical consequence that is clearly a forerunner of that in Tarski [1935] 1936. Ajdukiewicz cited its close relationship to the usage in Mario Pieri 1906g.
- Moreover, in an earlier paper, Ajdukiewicz had cited a precedent in the work of Bernard Bolzano a century earlier, which was being discussed at length in Lwów at this time (see Betti 2006).
- Tarski didn't cite Ajdukiewicz, but probably got some of his ideas about logical consequence from this close colleague, and thus indirectly from Pieri and Bolzano.
- Pieri's work was an earlier milestone. How did Pieri arrive there?

Mario Pieri

- Chiesa San Giusto, from Pieri's front steps in Lucca (pop. 22,800), where
- he was born in 1860, the
- 3rd of 8 children.
- His father was a lawyer.
- Pieri was schooled in Lucca and Bologna.

- Scuola Normale Superiore, Pisa (20 km from home)
- 1884 PhD (w. Luigi Bianchi) Theses:
  - *Singularities of Jacobians of 4, 3, 2 Surfaces*
  - *Studies in Differential Geometry*

Pieri in Turin, 1886–1900

← *Mole Antonelliana*

← *Corso San Maurizio*

← *Military Academy* (professor)

← *University* (assistant)

← *Via Verdi* (*Via della Zecca*)

← *Castello* (1934 photo)

Algebraic Geometers

Enrico D'Ovidio

⇒

Corrado Segre

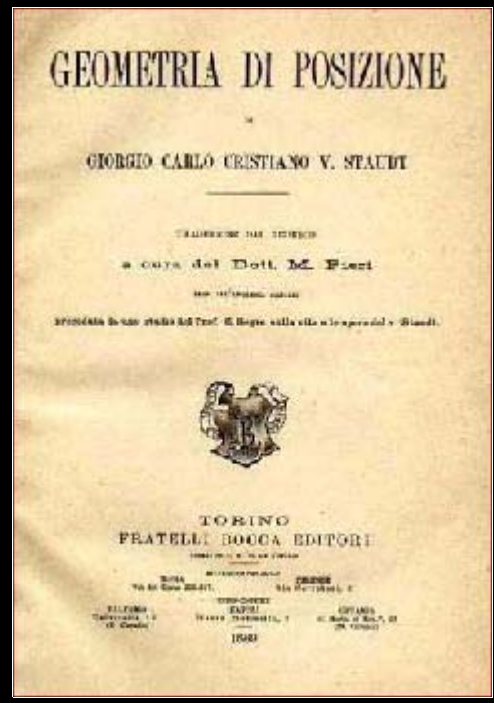
Mario Pieri

Segre, Pieri, and Peano

- In [1887] 1997, Segre asked Pieri to translate **Staudt's seminal 1847 work**.
- Pieri 1889a became a basic Italian resource.
- In 1891a Segre commented,

  There has not been established nor discussed…any system of independent postulates … [for] linear space of *n* dimensions, from which one can deduce the representation of its points by coordinates. It would be advantageous if some young scholar should occupy himself with this question.

- Pieri undertook that task and became a major participant in the school of foundations researchers led by his Turin colleague **Giuseppe Peano**.
- Peano emphasized the formulation of postulates, but only the negation of logical consequence: that is, their logical independence. And he was turning to other concerns.
- During 1895–1913 Pieri produced 23 works on logic and foundations of mathematics (13 on projective geometry).


GEOMETRIA DI POSIZIONE

GIORGIO CARLO CRISTIANO V. STAUDT

a cura del Dott. M. Pieri

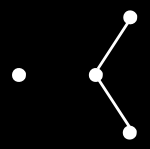
TORINO
FRATELLI BOCCA EDITORI

Giuseppe Peano

Pieri's Legacy

Our Book 1 (2007)

Axiomatics of Elementary Geometry:
***Point & Sphere* (1908a)**



➜ **Tarski ([1927] 1929, [1957] 1959)**

Axiomatics of $\mathbb{N}$: *On the Axioms of Arithmetic* (1907g)

Biography

Geometry: Foundations ➜ *Our Book 2* (2021)

Axiomatic Geometry

Projective: *Geometry of Position* (1898c)

➜ **Whitehead (1906)**

Elementary: *Point & Motion* (1900a)

➜ **Bachmann (1937, 1959)**

Arithmetic: Foundations

Philosophy of Mathematics ➜

Axiomatics: *Paris Lecture* ([1900]1901)

abstract systems
logical consequence

➜ **postulate theorists**
Ajdukiewicz (1921)
Tarski ([1935]1936)

- ***Pieri's legacy also includes***
  - **differential geometry: 3 research papers**
  - **vector analysis: 3 research papers**
  - **algebraic geometry: 6 PhDs (at Catania)**
    28 research papers and an edited book

Pieri's Impact on Foundations and Philosophy of Mathematics

- Clearly **formulating** and **describing** the axiomatic method
  - — Including a concept of **logical consequence** sufficient to support studies of consistency and independence of postulates, and
  - — and careful analysis of the role of **definitions**.
  - — But without today's emphasis on formalization of language.
  - — Acceptance was gradual and general but without acknowledging Pieri.

- Demonstrating our freedom to select **undefined notions**, and the construction of precise **definitions** involving them.
  - — Examples on the previous slide, with accounts of their impact.
  - — **Also** projective geometry based on • ↗ (1898b), and

    based on  (1901b), and

    complex projective geometry based on •  (1905c), and

    inversive geometry based on • ○ (1911d, 1912c).

    These have received only minor attention until recently.

- Restricting postulates to **1st-order** when possible, postponing simplification. Compare Pieri's and Tarski's axiomatizations of elementary geometry (see Tarski & Givant 1999).

Thank you for
your interest!

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Pieri, Tarski, and Axiomatics

James T. Smith

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