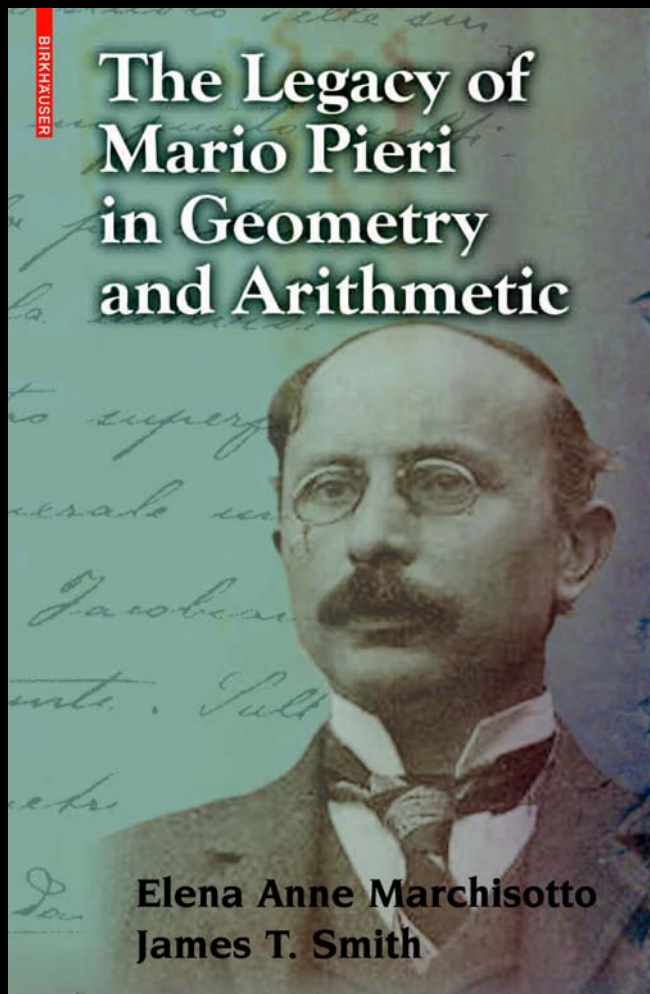


**Forty-Eighth Annual Conference: History of Mathematics
Miami University**

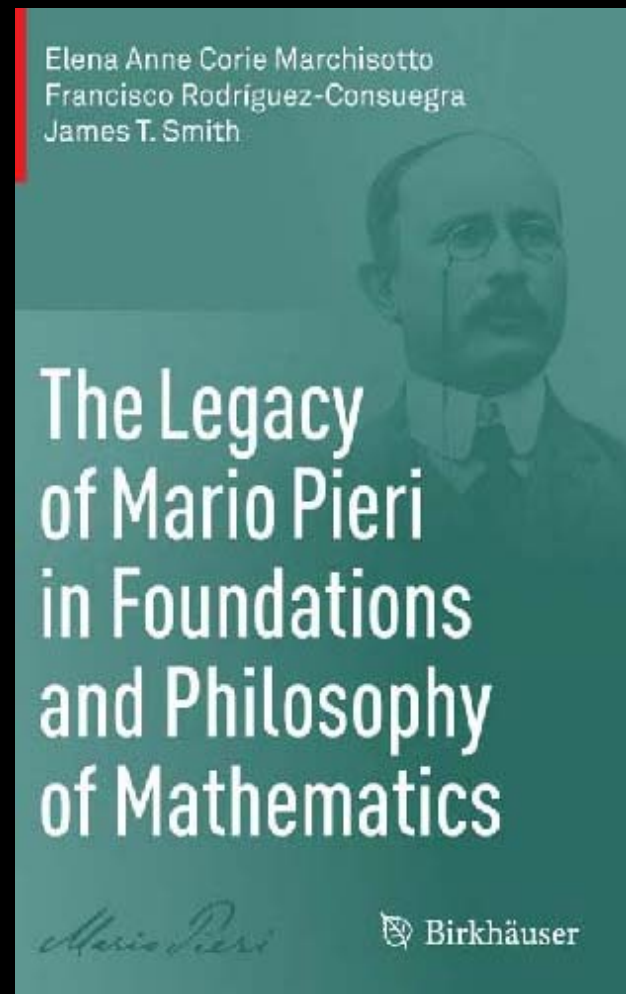
23–24 September 2022

Continuing Work on Mario Pieri (1860–1913)

**James T. Smith, Professor Emeritus
San Francisco State University**



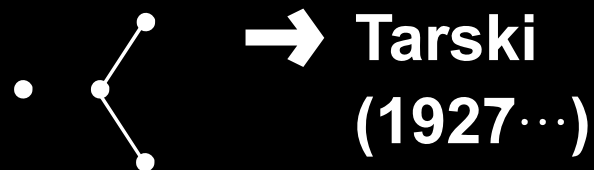
Birkhäuser, 2007



Birkhäuser, 2021

Book 1

Axiomatics of Elementary Geometry:
Point & Sphere
(1908a)



Axiomatics of \mathbb{N} :
On the Axioms of Arithmetic (1907)

Biography

Pieri's Legacy

← **Geometry:
Foundations** →

← **Arithmetic:
Foundations**

**Philosophy of
Mathematics**

⋮

(more later)

Book 2

Axiomatic Geometry
Projective: *Geometry of Position* (1898c)
• / → Whitehead
(1905)

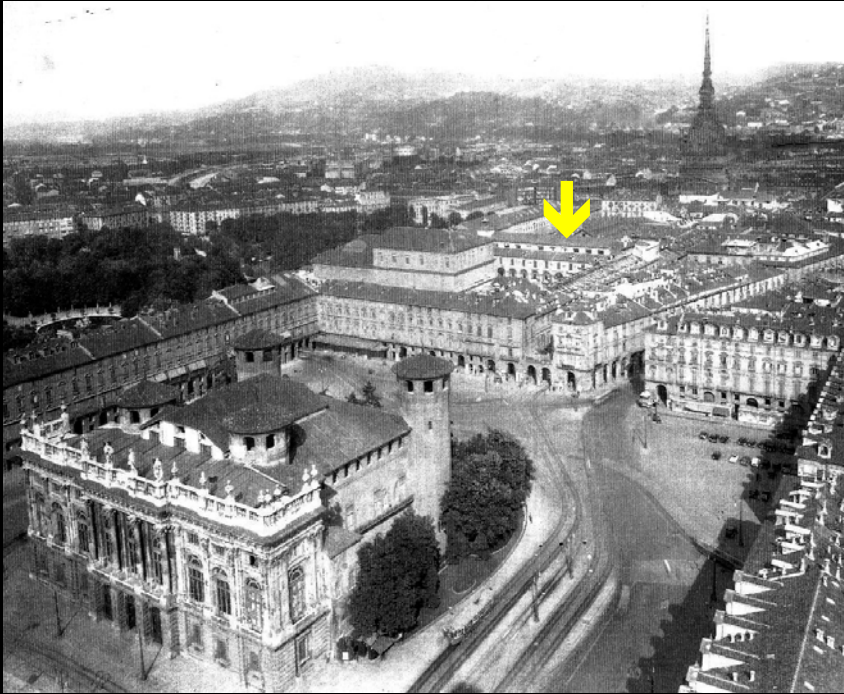
Elementary: *Point & Motion* (1900)
• ↗ → Bachmann
(1937, 1959)

→ Axiomatics: *Paris Lecture* ([1900]1901)
abstract systems
logical consequence
→ Tarski ([1935]2002)



- ***Chiesa San Giusto***, from Pieri's front steps in Lucca, where
- he was born in 1860, the
- 3rd of 8 children.
- His father was a lawyer.
- Pieri was schooled in Lucca and Bologna.
- ***Scuola Normale Superiore***, Pisa (20 km from home)
- 1884 PhD (w. Luigi Bianchi)
- Thesis 1: *Singularities of Jacobians of 4, 3, 2 Surfaces*
- Thesis 2: *Studies in Differential Geometry*

Pieri in Turin, 1886–1900



← *Mole Antonelliana*

← *Corso San Maurizio*

← *Military Academy*

← *University*

← *Via Verdi (Via della Zecca)*

← *Castello*

(1934 photo)

Algebraic Geometers



Enrico D'Ovidio

⇒

Corrado Segre

Mario Pieri

Segre and Pieri, I

- Segre's 1884a dissertation *On Quadrics in a Linear Space of Arbitrary Dimension* was “a **fundamental starting point** for the development of Italian projective n -dimensional geometry” (Brigaglia 1996).
- But Pieri had been solving very **narrow problems**. For example, in 1886c, *On the [# of] Double Normals to a Twisted Algebraic Curve*
$$= \frac{1}{2}(r + n)(r - 1) - \frac{1}{2}(n + m - \theta) + h$$
, where
 r, n, m, h = rank, order, class, # of apparent double points
(nomenclature from the JFM review by Gino Loria in Turin, another former student of D'Ovidio).

Segre and Pieri, II

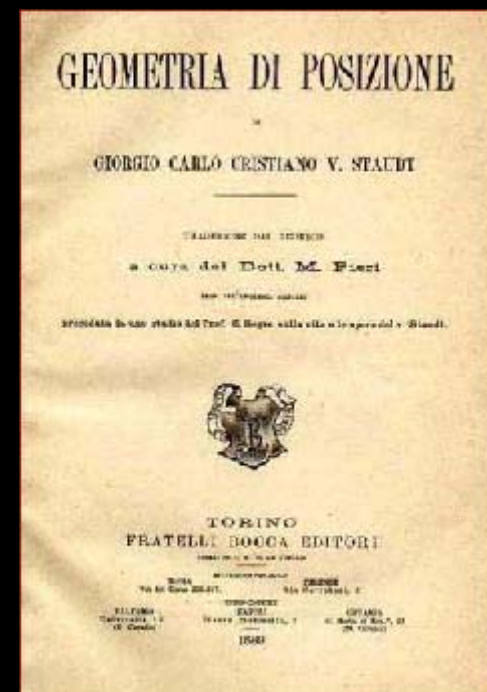
- Pieri used a **correspondence principle** about the relation between pts X, X' on a given line that holds when a homogeneous polynomial $F(X, X') = 0$:

$$(\forall X)(\exists \alpha' \text{ such } X') \ \& \ (\forall X')(\exists \alpha \text{ such } X) \Rightarrow (\exists \alpha + \alpha' \text{ pts } X) \ F(X, X) = 0.$$

- Pieri **generalized** this in 1887b, *The Correspondence Principle in an Arbitrary n -Dimensional Linear Space*, about the relation between pts X, X' of a given n -dimensional space that holds when n such polynomials $F_j(X, X') = 0$. His derived formula is more complex.
- In 1888 Pieri **used that formula** to give the first n -dimensional generalization of Bézout's theorem about the number of intersections of two algebraic curves. See Marchisotto 2021.

Segre, Pieri, and Peano

- In [1887] 1997, Segre asked Pieri to translate **Staudt's seminal 1847 work**:
- Pieri 1889a became a basic Italian resource.
- In 1891a Segre commented,
“There has not been established nor discussed... any system of independent postulates ... [for] linear space of n dimensions, from which one can deduce the representation of its points by coordinates. It would be advantageous if some young scholar should occupy himself with this question.”
- Pieri undertook that task and became a major participant in the school of foundations researchers led by his Turin colleague **Giuseppe Peano**.
- During 1895–1913 Pieri produced 23 works on logic and foundations of mathematics (13 on projective geometry).
- These are treated in depth in our two books.



Giuseppe Peano

Pieri's Legacy (Continued)

Algebraic Geometry
1884–1905

Differential Geometry
1884–1893

Vector Analysis
1912

(more later)

Needing Study

Thesis 1

Tangents, normals: 7 papers

Enumerative: 16 papers,
posthumous book

Birational transforms: 5 papers

PhDs guided (1902–1905): 6

Thesis 2

Research: 3 papers

3 papers, in part related to
Cesare Burali-Forti

Questions for *You* to Study

Algebraic and Differential Geometry

1. How do these works ***fit in*** with what others were and had been doing?
2. What did Pieri ***have to learn*** in order to conduct this research?
3. What was its ***impact***, immediate and longer-range? Especially, that of the enumerative geometry papers that gave rise to the ***Pieri formulas*** now in common use?
4. Exactly how did those formulas ***emerge***?
5. What, precisely, was Pieri's contribution to the [1915] 1991 French version *Méthodes énumératives* of the big *Encyklopädie* article by H. G. Zeuthen?

Vector Analysis

6. How was Pieri's work related to competing, more successful, approaches? What was his relationship with Burali-Forti?

A Teaching Legacy: Surviving Materials

Projective Geometry

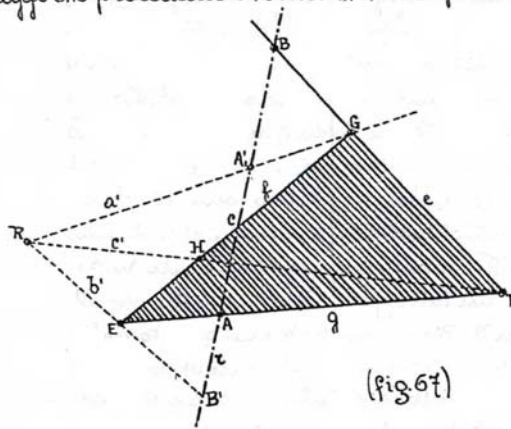
1891c published Turin lectures

1910 handwritten Parma lectures

292

Proposizione fondamentale. Le tracce dei lati di un triangolo sopra una retta arbitraria del suo piano, che non passi per nessun vertice del triangolo, sono in involuzione coi tre raggi che proiettano i vertici opposti a quei lati da un punto del piano del triangolo, il quale non giaccia su nessun dei tre lati.

Sia EFG il triangolo, r la retta e R il punto dati nel piano del triangolo. Indichiamo con A, B, C rispettivamente le tracce di EF, FG, GE sopra r , e con a', b', c' i raggi che proiettano i vertici G, E, F dal punto R .

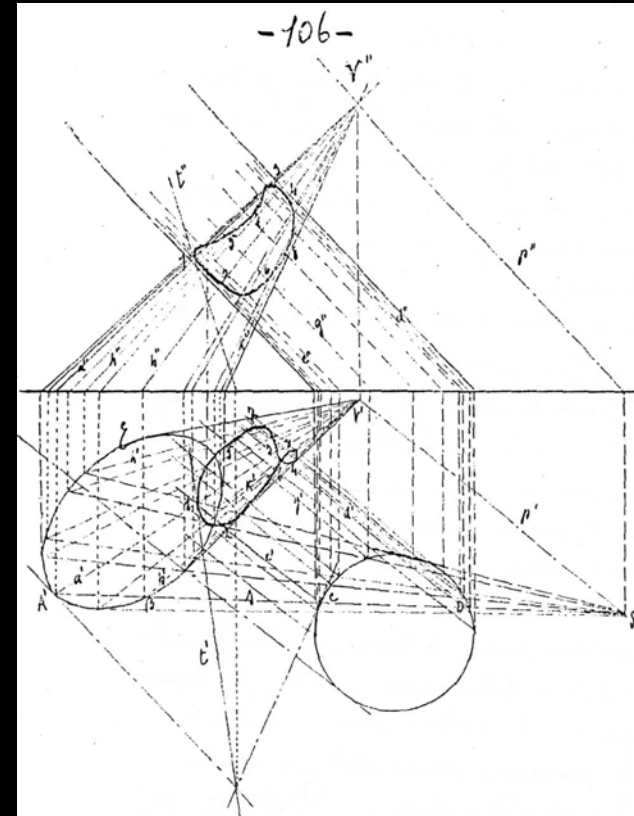


(fig. 67)

Si deve provare che A, B, C ed a', b', c' sono in involuzione.

Descriptive Geometry

1912f handwritten Parma lectures



← 200 days' work by assistants!

7. How are Pieri's methods related to others'?

I hope ***some younger historians*** will pursue these questions and report their findings!

**Thank you for
your interest!**

**James T. Smith
Professor Emeritus
San Francisco State University**

Continuing Work on Mario Pieri (1860–1913)

James T. Smith

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