Exercises on exponents and radicals

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You should become fairly adept at simplification of algebraic expressions involving exponents and radicals, because they occur unannounced in the middle of more meaningful problems. If you get hung up on them, you'll never get to the good stuff. These exercises are selected from my grandfather Bert Smith's copy of a text that was popular in his time: William J. Milne, *The inductive algebra, embracing a complete course for schools and academies*, New York: American Book Company, 1881. I don't know if he completed high school, but he was a schoolteacher in Champaign County, Ohio, around 1895. Milne's is the algebra text I paid most attention to as a student. Strive for simplest forms of these expressions. Suggested solutions are included. Some of these may have more than one optimal form, so part of each exercise is to verify that your solution is equivalent to mine.

1. $\dfrac{27x^3y^2z^2}{-9x^2y^2z} =$

2. $\dfrac{1}{a^2x^{-2}y^{-3}} =$

3. $\dfrac{20a^{-4}b^3c^{-3}}{-5a^{-2}b^6c^{-5}} =$

4. $(a^4b^{-2}c^{-3})^8 =$

5. $(2a^{-2}b^2c^{1/3})^9 =$

6. $\dfrac{5x^2y^2}{a^2x^2}\cdot\dfrac{3ax^2}{2ay^2} =$

7. $\dfrac{\dfrac{mny^2}{abc}}{\dfrac{m^3n^2y^2}{a^2b^2c^2}} =$

8. $\left(\dfrac{a^{-2}b^n}{c^{-4}d^3}\right)^6 =$

9. $\sqrt[4]{16a^4b^8c^8} =$

10. $\sqrt[5]{-a^5c^{10}x^2y} =$

11. $\sqrt[3]{\dfrac{125x^3y^3}{216a^6y^3}} =$

12. Express $3b\sqrt{bxy}$ entirely under the radical.

13. Express $(6a^2b^3)^{2/3}$ with a radical.

14. $a^{1/2}\cdot a^{1/4} =$

15. $2\sqrt{12}\cdot 3\sqrt{6} =$

16. $4\sqrt[3]{2x^2y^2}\cdot 5\sqrt[3]{4x^2y} =$

17. $\sqrt[3]{axy^2}\cdot\sqrt{3axy} =$

18. $a^{1/3}/a^{1/2} =$

19. $\dfrac{6\sqrt{54}}{3\sqrt{2}} =$

20. $\dfrac{\left(4x^2\right)^{1/3}}{\left(2ax\right)^{1/2}} =$

21. $\left(4a\sqrt[3]{3b^2}\right)^2 =$

22. $\left(2a\sqrt{3a}\right)^3 =$

23. $\sqrt[4]{a^3b^3\sqrt[3]{a^2b^2}} =$

Solutions

1. $-3xz$

2. $x^2 y^3/a^2$

3. $-\dfrac{4c^2}{a^2 b^3}$

4. $\dfrac{a^{32}}{b^{16} c^{24}}$

5. $512\, b^{18} c^3/a^{18}$

6. $^{15}\!/\!_2\, (x/a)^2$

7. $\dfrac{abc}{m^2 n}$

8. $\dfrac{c^{24} b^{6n}}{a^{12} d^{18}}$

9. $2 a b^2 c^2$

10. $-a c^2 x^{2/5} y^{1/5}$

11. $^5\!/\!_6\, x/a^2$

12. $\sqrt{9\, b^3 xy}$

13. $\sqrt[3]{36\, a^4 b^6} = ab^2 \sqrt[3]{36a}$

14. $a^{3/4}$

15. $36\sqrt{2}$

16. $40 x y \sqrt[3]{x}$

17. $a^{5/6} x^{5/6} y^{7/6} \sqrt{3}$

18. $a^{-1/6}$

19. $6\sqrt{3}$

20. $2^{1/6} a^{-1/2} x^{1/6}$

21. $16 a^2 b^{4/3} \sqrt[3]{9}$

22. $24\, a^{9/2} \sqrt{3}$

23. $(ab)^{11/12}$